# Pascual Jordan's legacy and the ongoing research in quantum field theory

dedicated to my teacher and role model: Rudolf Haag

appeared in "Historical Perspectives on Contemporary Physics" Eur. Phys. **35**, J.H.

377 (2010)

Bert Schroer
present address: CBPF, Rua Dr. Xavier Sigaud 150,
22290-180 Rio de Janeiro, Brazil
email schroer@cbpf.br
permanent address: Institut für Theoretische Physik
FU-Berlin, Arnimallee 14, 14195 Berlin, Germany

December 2009

**Abstract**

Pascual Jordan's path-breaking role as the protagonist of quantum field theory (QFT) is recalled and his friendly dispute with Dirac's particle-based relativistic quantum theory is presented as the start of the field-particle conundrum which, though in modified form, persists up to this date. Jordan had an intuitive understanding that the existence of a causal propagation with finite propagation speed in a quantum theory led to radically different physical phenomena than those of QM. The conceptional-mathematical understanding for such an approach began to emerge only 30 years later. The strongest link between Jordan's view of QFT and modern "local quantum physics" is the central role of causal locality as the defining principle of QFT as opposed to the Born localization in QM.

The issue of causal localization is also the arena where misunderstandings led to a serious derailment of large part of particle theory e.g. the misinterpretation of an infinite component pointlike field resulting from the quantization of the Nambu-Goto Lagrangian as a spacetime quantum string.

The new concept of modular localization, which replaces Jordan's causal locality, is especially important to overcome the imperfections of gauge theories for which Jordan was the first to note nonlocal aspects of *physical* (not Lagrangian) charged fields.

Two interesting subjects in which Jordan was far ahead of his contemporaries will be presented in two separate sections.

## Contents



# 1 Preface

Some years ago Jürgen Ehlers (1929-2008), Jordan's first postwar PhD student and the founding director of the AEI in Golm, asked me to use my expertise of quantum field theory (QFT) to check out some papers by Pascual Jordan with a seemingly somewhat inaccessible content which appeared to have been left out in existing biographies and accounts of his scientific legacy. It took me several years to fully appreciate their content and to understand that they were early harbingers of subtle quantum field theoretic properties which, in a somewhat different context and a broader setting, had their comeback in the QFT of the 60s and 70s; in the case of the 1924/25 Einstein-Jordan conundrum one even needs the recent insight about the connection between modular localization, vacuum polarization at the localization boundary and thermal aspects from the restriction of the vacuum state to localized algebras; in other words the full understanding of the QFT model with which everything begun requires the knowledge of the most advanced notions of local quantum physics.

I included some of these results in a talk with a large biographical part which I presented under the title: "Pascual Jordan, biographical notes, his contributions to quantum mechanics and his role as a protagonist of quantum field theory" at a conference dedicated to the memory of Pascual Jordan 2005 in Mainz, Germany whose written account can be found online [1]. Here the biographical material is left out and the presentation of Jordan's QFT contribution will be extended.

As a quantum field theorist I am intrigued by the modernity of Jordan's view of the subject. In his contribution in the famous 1925 Dreimaennerarbeit, which is nowadays considered as the cradle of QFT, Einstein's at that time still controversial photon ("Nadelstrahlung") in terms of thermal fluctuations in a subvolume was carried forward into the new quantum theory with the help of a subinterval in a two-dimensional (conformal) free quantum "photon" field. The problem of subinterval fluctuation was treated in terms of quantum mechanical infinitely many oscillators which, as we know nowadays, does not adaequately describe the holistic quantum field theoretic fluctuation problem related to localized subvolumes. But already in his 1928 Habilitationsschrift and in his famous paper with Pauli, the property of causal locality as the characterizing principle of QFT took the center stage and the fundamental difference between QFT and Dirac's idea of a relativistic quantum mechanics (QM) begun to be appreciated. My interest to write a paper about to what extend modern QFT has vindicated Jordan's ideas arose in this context.

What started as a friendly turn to my highly regarded senior colleague (and mentor during my brief participation in Jordan's seminar around 1956), the late Jürgen Ehlers, changed into surprise when I looked at some of the insufficiently understood publications of Jordan. It became gradually clear to me that these papers constitute early flare-ups of the intrinsic conceptual logic of QFT whose correct mathematical understanding and physical interpretation was a task beyond the level of QFT at the time of Jordan: even nowadays those issues would hardly be accessible without foundational knowledge about local quantum physics; this explains in a way why those contributions have been left out in presentations of Jordan's oeuvre by other authors.

Already at that time of the Mainz conference I thought that it would be interesting to present some of Jordan's work written in the years 1930-38 which was lost in the maelstrom of history[1] in more details, in order to reveal its startling modernity. Though as a passionate champion of research on the frontier of QFT, I did not want to subject myself to the time-consuming rules of documenting historical events, but I find the task of creating conceptual and philosophical connections to ideas which, either as a result of world war II, or through other sociological reasons, were lost or took a different turn, quite challenging. In fact there is no better critical view of ongoing particle physics than that obtained from comparing the present state of affairs with what the protagonist of QFT expected from his brainchild.

With the launch of the new Springer journal EPJ H, dedicated to the historical illumination of actual research problems, it is now possible to recreate some of these lost ideas and demonstrate how deep thoughts occasionally flare up and disappear for some time before they firmly and permanently assert themselves in the actual research**.** Ideas which have no continuous link to present day QFT, or which led to profound errors are presumably not of much interest to historians, but to the extend that they have left traces which are important for the

[1]These were the years in which according to a remark by Peter Bergmann a publication in Zeitschrift fuer Physik "was like a first class burial".

future of particle theory, they should be presented next to the successes. The present paper is strictly limited to the subject in the title and in the abstract. For biographical and historical details I refer to a previous paper [2]. A rather complete bibliographical list has been compiled in [3].

## 2 The first phase of Quantum Field Theory

Quantum field theory (QFT) is often referred to as relativistic quantum mechanics (QM), but this characterization has no convincing conceptual or historical justification. Relativistic QM exists, but the conceptual as well as computational unfolding of QFT has increased the distance between the two. The conceptual-mathematical meaning of "relativistic quantum mechanics" (QM) in the discourse between Jordan and Dirac was somewhat vague, but we know by now how a consistent relativistic quantum mechanics of interacting particles looks like, and it has no similarity with interacting QFT besides the fact that both theories are quantum theories (QT) and carry a unitary representation of the Poincaré group (section 3.1).

Historically the birth of field quantization dates back to Jordan's contribution to the 1925 "Dreimännerarbeit" [4] which consisted in the first quantum theoretic derivation of Einstein's fluctuation formula, thus proving that presence of the wave term and the particle-like (photon) contribution on which Einstein [7] based his photon hypothesis long before the latter was observationally verified in experiments by Compton as well as by Bothe and Geiger. Hence QFT, which gave rise to many new problems, originated itself at the same time as QM in the context of a solution of a problem posed by Einstein. But even at the time of the publication of the BHJ paper, as well as during all the time since Einstein's famous 1905 "photon" paper (confirmed and strengthened in his 1917 work [7]), there remained a certain reservation against Einstein's indirect pro photon arguments for which his reasoning based on the fluctuations in a finite volume formula was his most forceful counter argument.

Even Jordan's two coauthors of the Dreimännerarbeit were not entirely convinced by his arguments which went against the, at that time still rather popular, theory by Bohr, Kramers and Slater who tried to do treat radiation by semiclassical methods in the Bohr-Sommerfeld setting i.e. without photons; these doubts lingered on for several years to come until that theory was abandoned. However serious doubts about quantum mechanical fluctuation being able to substitute Einstein's thermal fluctuation and limit the frequency summation just to avoid divergencies. As Duncan and Janssen recently recalled [5] (and the present author certainly agrees with, despite some critical later remarks), Jordan's field theoretic model in support of a quantum theoretical derivation of Einstein's thermal fluctuation formula, despite some shortcomings, really heralded the beginning of QFT. However the concepts and methods which are needed in order to show that the restriction of the pure vacuum state to a subinterval generates an impure state of a thermal kind (not possible in QM!) which is related to Einstein's statistical mechanics fluctuation calculation

is an insight into QFT which are of more recent origin [6] (more in section 4). For this task one needed to understand the thermal aspect of modular localization which generalizes the well-known observations which Hawking and Unruh made in specific cases about thermal aspects of quantum matter behind or in front of black hole horizons or behind causal horizons associated with a Rindler wedge region. The modular localization theory permits to show that these vacuum fluctuation-caused effects from localization to subregions really explains the thermal behavior as derived by Einstein from pure stastisical mechanics considerations.

Ironically the strongest argument that this model represents genuine QFT and not some infinite degrees of freedom second quantized QM (as the work of Jordan and Klein [55]) is the fact that neither Jordan nor anybody afterwards was able to present a rigorous pure quantum mechanical derivation about a free field theory without invoking additional assumptions and plausibility arguments at some places. The restriction of a quantum mechanical vacuum to a subinterval causes a tensor-factorization of the Hilbert space and the operator algebra. In QFT this changes radically: there is no inside-outside tensor factorization[2] in any state and the vacuum reduced to the interval is highly impure [16]. QM can never lead to a impure thermal state merely by restricting the vacuum to a subinterval; the Born-localization related to eigenfunctions of the position operator has no thermal or impure aspects, rather it shares its concepts of entanglement with quantum information theory. With (uncontrolled) additional assumptions which destroy the holistic localization property of QFT, QM yields a formula consisting of a wave and a particle contribution [4], but it is not really capable to fully solve the Einstein-Jordan conundrum, since the explaining quantum theory should identify the reduced vacuum state as a mixed state with thermal properties. QFT on the other hand accomplishes this magic. We know about this thanks to many decades of research in local quantum physics which has led in particular the recent insights into *modular localization* [16][66][6].

Quantum mechanical methods of mode decomposition and maintaining finiteness by occupying certain modes are generally not correct since they destroy causal localization; there is hardly any bigger conceptual-mathematical contrast as that between the Born-localization of QM and the modular localization of QFT [16]. The latter forces the theory to behave in a much more holistic way than QM. Even though the discovery of QFT took place within the same year as QM, the elaboration of the computational and conceptual instruments to understand and explore it has lasted already for more than 80 years and is still not anywhere near its closure (more remarks in section 4), whereas the principles of QM were fully understood within less than two decades after their discovery.

Actually the prelude to this most fascinating episode in the history of quantum physics started already with Jordan's thesis [8] which he defended and

[2]Hence also the notion of entanglement losses its meaning. Only if one prevents the localization region from touching its causal complement by the "splits" procedure one starts to control Heisenberg's boundary vacuum polarization clouds and returns to a tensor factorization, but the reduced split vacuum state remains impure and thermal.

submitted to Zeitschrift für Physik in 1924, i.e. the year before the appearance of the famous work of Heisenberg on quantum mechanics (QM) and the Dreimännerarbeit. The topic of his thesis was a quantum theory of radiation, still without photons, or in the jargon of the pre-photon times, without Einstein's "Nadelstrahlung" (the result of the photon recoil). By submitting his thesis for publication with Born's blessings (besides Einstein and Pauli there were no strong defenders of the photon hypothesis), Jordan [8] set out to openly contradict Einstein [9] by showing that the existence of Nadelstrahlung was not necessary in order to establish thermodynamic equilibrium. He did not have to wait long for an equally forceful reaction. Einstein conceded that there was nothing wrong in Jordan's mathematics, but that in a pure wave picture without the (photon) Nadelstrahlung the calculation of the correct radiation absorption coefficients cannot be obtained.

This episode was Jordan's figurative road to Damascus as far as his conversion to the existence of photons was concerned. Even if, as Born and especially Heisenberg believed, Jordan did not completely solve the fluctuation conundrum, he presented us with the beginnings of the richest theory of quantum matter up to date: QFT. By the time he wrote his section in the Dreimaennerarbeit he had given up the attempt to obtain thermal equilibrium without photons and became the radical annunciator of a QFT of waves, including the de Broglie matter waves. He took great pride about his ability to explain Einsteins fluctuation conundrum, even if his colleagues maintained some reservations. Being strongly philosophically oriented, he could not tolerate a wave quantization for light and quantum mechanics for matter, it had to be a unified setting for both. His intention to draw Einstein towards this new conceptual setting was however less successful; Einstein had a lot of praise for Jordan's abilities, but he never subscribed to QFT or QM as the final quantum matter theory; the philosophical aspects which attracted Jordan (the probabilistic aspects of quantum theory was no problem for him) were exactly those which did not find the likings of Einstein, although both held the universality of physical principles in high esteem.

The friendly competion with his adversary Dirac, who up to the beginning of the 50s upheld the viewpoint that matter should be described in terms of infinitely many quantum mechanical oscillators and the wave quantization should be reserved for light (just as in classical wave/particle theory), was more substantial because it took place between two individuals who fully accepted quantum theory; in fact it was of tremendous benefit to the development of QT, both for the particle as well as the field side. Even those discoveries of Dirac which later revealed themselves to be inconsistent on a deeper level[3], as the particle-inspired hole theory, came together with gems of permanent endurance as the Dirac equation. The latter was instrumental in Wigner's representation theoretic analysis [12] of relativistic one-particle states, the first completely intrinsic

[3]The inconsistency onely showed up in higher order perturbation theory involving vacuum polarization. They did not affect the computation in Heitler's [11] and Wenzel's book, but it is not possible to formulate renormalization theory in this charge-unsymmetric setting. The repair of this theory leads to QED.

relativistic QT (without any quantization parallelism).

To be conceptually and mathematically precise on this point of QFT versus QM, one should add that relativistic QM really exists as a automomous consistent theory (the inconsistency of Dirac's hole theory did not dispel it); but in its conceptual-mathematical details it is quite different from what Dirac thought is should be. The relativistic aspect (representation of the Poincaré group) formulated in terms of particles leads to a multiparticle representation which, in the presence of interaction potentials, cannot be written down by adding up pair potentials (as in the nonrelativistic QM where the cluster factorization of the n-particle factorization and the S-matrix are obtained without any effort). Rather the 3-particle interaction has to be determined from the two-particle potential by an interplay of relativity and cluster arguments and not by simply adding up two-particle potentials; this "cluster" construction has to be inductively implemented.

Since in the step from n to n+1 particles the upholding of the cluster property demands the introduction of a connected "minimal" n+1-particle interaction potential which is induced in terms of the previously determined interaction potentials, there is not intrinsic distinction between elementary interaction potentials and induced ones. This "direct particle interaction" (DPI) (i.e. not field mediated) [17], unlike the Schrödinger QM, does not admit a natural "second quantization formalism"[4], even though its multiparticle Poincaré group representations cluster-factorize.

The existence of such relativistic particle theories (called direct particle interactions (DPI)), which satisfy all the conceptual properties which one can express *in terms of particles without using interpolating fields*, is not very interesting by itself; but by creating a sharp contrast to the causal locality of QFT it his of invaluable help in understanding the subtleties of the latter. Most of the serious errors committed in the last 5 decades, in particular the misunderstanding that string theory has something to do with spacetime localized strings and that matters of localization can be red off from the S-matrix, result from misconceptions about QM and QFT localization; in fact all differences between these two quantum theories, which, as mentioned, do not disappear by making QM relativistic, originate from their different localization, as will be shown in the next section.

Since interactions (which in QFT would be fixed in terms of local coupling parameters), need potential functions instead of coupling parameters in order to be defined, the DPI setting is certainly not "fundamental"[5]. Its very existence permits to immediately refute some conjectures concerning the relation between particles and fields. In particular Weinberg's idea that the unitarity, Poincaré invariance and cluster factorization of the S-matrix can only originate in a QFT setting [13] is not correct, it remains even incorrect if one adds to the list the timelike form of macro-causality (Stueckelberg's "causal re-scattering").

[4]This is of course related to the fact that the cluster factorization is not the result of the additivity of interaction potentials.

[5]Nuclear physicists use the DPI setting to decribe elastic scattering (or the creation of only a few particles) in intermediate energy regions.

Of course Weinberg (as well as the present author) would never consider DPI as a fundamental relativistic QT; but for showing the sublety of causal localization (which cannot be expressed in terms of particles and their clustering) it is an ideal surgeons knife which separates the particle from the field body. In the present context a rudimentary conceptual knowledge of DPI is valuable because it serves to lift the field-particle relation, personified by Jordan and Dirac, to a higher level than it has been discussed in most historically motivated articles. In contrast to the particle-wave conundrum of the beginnings of QT, which disappeared with the understanding of the equivalences between different formulation of QM and the ascend of QFT, the field-particle relation, despite progress, has not yet been closed.

In fact since the more than 80 years of its existence of QFT it was always with us, and the attempts to get rid of it by adopting a pure S-matrix view failed in each attempt, and even nowadays there is a large community which was hitherto unable to liberate itself from the self-knitted webb of conceptual confusions. Although there has been great progress, there is as yet no closure on this issue; in fact it appears as if the decisive nonperturbative understanding concerning an intrinsic classification, as well as an existence proof and a construction of models is still to come[6]. So a large part of this presentation of Jordan's QFT uses history as a pretense to lure the historically educated interested reader into the ongoing "theoretical laboratory", or in the other direction, to reveal to the professional quantum field theorist the ideas which marked the beginning of QFT. The aim is always to get a new perspective about the the relation of the ongoing research and its history. This is also the best way to acquire some immunity against the incorrect idea that QFRT arrived at its closure and is on its way of becoming a footnote to the end of the millennium's "theory of everything".

Jordan in his search for the basic principles which underlie QFT did not stay with a quantization of oscillators setting as in the Dreimännerarbeit. In his Habilitationsschrift [14] 1927 he left no doubt that the crucial point was not the quantization of a collection of oscillators, rather QFT was the promotion of the causal locality principle of Maxwell's action at the neighborhood which defines propagating classical field theory, to the realm of quantum physics.

The distinction between the localization in QM based on Born's probability applied to Schrödinger wave functions, and on the other hand the causal localization which is intrinsic to QFT without a directly associated probability, is more radical than it appears at first sight. It accounts for the significant difference between fields and particles in the presence of interactions which remain unrelated within finite spacetime regions, but become interwoven in large timelike regions through scattering theory. With other words one needs the S-matrix to relate fields with particles, a relation which for its mathematical formulation requires to study suitably defined asmptotic limits of quantum fields. But even in the absence of interactions, when particles and fields are directly related even

[6]This goal has been achieved to some degree in d=1+1 (chiral models, factorizing models) based on modular localization in the setting of operator algebras.

in finite spacetime regions, it would not be sufficient to tell a newcomer who just learned QM and knows how to deal with arrays of uncoupled oscillators, that a "free field is nothing but a collection of infinitely many harmonic oscillators"; he would not know how to combine the infinite array of oscillators to a holistic object which is localized in spacetime. To understand this holistic aspect took many years from the inception of QFT in the Dreimännerarbeit to the Jordan-Pauli work on causal commutator functions. And in the presence of interactions the analogy with oscillators has anyhow no conceptual content.

Initially one may have thought that the distinction between QM and QFT is just the existence of a maximal velocity in QFT (important for the notion of micro-causality) compared to its absence in QM. As the example of acoustics shows, special models of QM have an asymptotic (large timelike distances) *effective* limiting velocity obtained as a quantum mechanical expectation value in appropriate states and depending on the kind of quantum mechanical matter (viz. the acoustic velocity in QM). It is precisely in this effective way that relativistic QM produces the velocity of light as an effective limiting velocity, which is sufficient to construct a Poincaré an invariant Møller operator and an S-matrix. Actually QFT employs both concepts, for the asymptotic particle wave functions the frame-dependent quantum mechanical localization related to the non-covariant position operator and the causal micro-locality to be used for the description of local observables and covariant fields. The latter played the crucial role in the derivation of the experimentally verified dispersion relations[7]. The appearance of particles in the asymptotic scattering limit of interacting theories turns out to lead to the *asymptotic coalescence of the probabilistic Born-Newton-Wigner localization with the modular localization of QFT*[8] *[16]* in which the modular localization inherits the probability and the the quantum mechanical BNW becomes covariant (independence of the frame). This is also the only place where one needs to maintain a connection between fields and particles. Without it QFT would just be an interesting mathematical structure with no observational consequences.

One can safely assume that Jordan, at the time of his 1927 Habilitation, knew some aspects of these propagation differences and their foundational consequences. The first formula which incorporates the causal localization structure of relativistic theories (as compares to the noncovariant equal time commutation relations which also hold in nonrelativistic theories) appears in his work with Pauli [56]. The modern point of view that the different models of QFTs are only different manifestations of a universal causal locality principle of which the QFTs which can be be obtained by Lagrangian quantization form only a small subset is an extension of that of Jordan but one which is separated by

[7]The adaptation of the optical Kramers-Kronig relations to the scattering theory of QFT was a very sophisticated rigorous theoretical project of the late 50s and early sixties; in fact it was the only research topic in particle physics which (together with is experimental verification) merits the characterization "mission accomplished".

[8]There is no fight of "Reeh-Schlieder (modular localization) wins against Newton-Wigner" [15] in large time asymptotic regions, rather only peaceful coexistence. In particular the asymptotic movement of particles can be described by velocity lines particle velocities being limited by the lightcone.

many decades of hard work and a sociologically caused loss of continuity with the first phase of QFT.

# 3 Fields versus particles: Jordan and Dirac

The issue of *particles versus fields* is at the heart of relativistic quantum physics and although amazing insights into this subtle issue have been obtained since Jordan, nobody with a profound knowledge of modern QFT would venture to say that it arrived at its conceptual closure. Without interactions, i.e. for free fields, the two concepts are closely related, but the presence of interaction-caused vacuum polarizations as a consequence of relativistic localization separates them significantly. Although n-particle states in the form of (anti)symmetrized tensor product of one-particle states under certain special (but important) circumstances span the Hilbert space and in this way grant the Hilbert space the form of a Wigner-Fock space, this particle structure is basically global. On the other hand if in a model of QFT for with a compact localized observable (i.e. a quantum field smeared with a finite supported test function) applied to the vacuum leads to a to a one-particle state, it is necessarily an interaction-free theory; or to phrase it in the opposite way, the presence of an interaction is inexorably accompanied by the presence of a vacuum polarization cloud (infinitely many particle/antiparticle pairs) near the boundary of the spacetime localization region inside which the desire one-particle state is localized. A similar shorter but more drastic way to say the same is: a local "bang" on the vacuum generates a vacuum polarization cloud.

The phenomenon of vacuum polarization was first seen with relativistic current operators by Heisenberg [18] and than generalized to interacting fields in the setting of perturbation theory by Furry and Oppenheimer [19]; in the Dreimännerpapier QFT was basically the quantization rules of QT applied to a chain of oscillators. It can be assumed that by 1930 Jordan had at least some intuitive knowledge of the importance of vacuum polarizations in connection with causal locality since he followed the development up to the middle of the thirties when the worry about his less than satisfactory position at a provincial university far from the ongoing quantum dialogue which he enjoyed in Goettingen and the frustration about his inability to draw some professional advantages from his support of the Nazi party begun to take his better part [1].

Before going into more details about the causal localization underlying QFT, it is appropriate to present the easier part: the Born-Newton-Wigner localization underlying QM and being the one which plays an important role in assigning Born probabilities to wave functions of particles which is even indispensable in QFT if it comes to the calculation of scattering cross section. Whereas for finite times there is a significant distinction between the frame-dependent Newton-Wigner localization (the Born-localization adapted to the relativistic form of the inner product) and the covariant causal localization defined through fields, their asymptotic large time compatibility is the precondition for QFT in order to be an observational accessible.

### 3.1 The position operator of QM, the frame-dependent particle localization and the covariance of asymptotic correlations

It is well-known that the localized states of Schrödinger QM are (improper) eigenstates of the position operator. They are frame-independent in the Galileian sense of nonrelativistic QM. One can formally adapt this to relativistic particle states (Newton-Wigner), but one looses the frame-independence and hence the existence of a covariant position operator. This localization appears in QFT only in an asymptotic sense, always in connection with particles.

The *direct particle interactions* (DPI) (where "direct" means "not field-mediated") is a relativistic theory in the sense of representation theory of the Poincaré group which, among other things, leads to a Poincaré invariant S-matrix. Every property which can be formulated in terms of particles (as the cluster factorization into systems with a lesser number of particles as well as other timelike aspects of macrocausality), can also be implemented in this setting. The S-matrix does however not fulfill such analyticity properties as the crossing [20] property whose derivation relies on the existence of local interpolating fields.

In contradistinction to the more fundamental locally covariant QFT, DPI is primarily a phenomenological setting, but one which is consistent with every property which can be expressed in terms of relativistic particles only. So instead of approximating nonperturbative QFT outside conceptional-mathematical control, the idea of DPI is to arrange phenomenological calculations in such a way that the "approximation" at least preserves the principles of relativistic mechanics and macro-causality i.e. of the only form of causality which can be formulated in terms of interacting particles without invoking fields which interpolate their incoming/outgoing asymptotic particles [17]. Although its practical use is limited to phenomenological settings for medium energy nuclear processes in which only a few mesons are created, it is ideally suited to understand what Dirac's relativistic particle view leads to (probably not what he had in mind) and it also helps greatly why macrocausality in terms of particles with an additional esthetical formal simplification led Stückelberg to Feynman rules before Feynman and before their field theoretic derivation. This shows that formally there is a close relation of the two settings, even though they are very far apart in their underlying physical principles.

As for any quantum mechanical prescription, the necessity to choose interaction functions (potentials V) instead of local coupling constants between fields as in QFT, already indicates that such a description is less on the fundamental and more on the phenomenological side. The characterization as "phenomenological" should however not be confused with "ad hoc rules", which often turns out to be the present day meaning of the word "effective" interaction. In fact the methods of DPI particle interactions leading to a unitary cluster-factorizing representation and a Poincaré-invariant scattering matrix S are highly sophisticated since relativity contradicts the naive implementation of the cluster factorization by simply adding pair interactions of the two-particle subsystems. Perhaps if

Dirac would have known these intricacies he would not have insisted in a particle based approach up to 1950 (when he finally took up QFT not only for photons but also for massive quantum matter). But then we would have missed all the wonderful discoveries he made under the pretense of relativistic particles which are not affected by these subtleties of multiparticle interactions.

Needless to add the DPI and QFT are the only conceptual frameworks of relativistic QT; DPI is a particle-based relativistic QM[9] which besides the relativistically invariant semi-global Møller operator and ensuing S-matrix contains no covariant objects at finite times (in particular no covariant position operators), and covariant localizable QFT which for implementing causal localization (related to the macroscopic maximal velocity of light) requires the prize of containing no localized interacting particles and admitting particle correlation probabilities only asymptotically.

For the interaction of two relativistic particles the introduction of interactions amounted to add to the free mass operator (the Hamiltonian in the c.m. system) an interact which depends on the relative position and momentum. The exigencies of representation theory of the Poincaré group are then fulfilled and the cluster property stating that $S \to \mathbf{1}$ for large spatial separation is a consequence of the short ranged interaction. Assuming for simplicity identical scalar Bosons, the c.m. invariant energy operator is $2\sqrt{p^2+m^2}$ and the interaction is introduced by adding an interaction term $v$

$$M = 2\sqrt{\vec{p}^2 + m^2} + v, \quad H = \sqrt{\vec{P}^2 + M^2} \tag{1}$$

where the invariant potential $v$ depends on the relative c.m. variables $p, q$ in an invariant manner i.e. such that $M$ commutes with the Poincaré generators of the 2-particle system which is a tensor product of two one-particle systems.

One may follow Bakamjian and Thomas (BT) [21] and choose the Poincaré generators in their way so that the interaction only appears in the Hamiltonian. Denoting the interaction-free generators by a subscript 0, one arrives at the following system of two-particle generators

$$\begin{aligned} \vec{K} &= \frac{1}{2}(\vec{X}_0 H + H\vec{X}_0) - \vec{J} \times \vec{P}_0 (M+H)^{-1} \\ \vec{J} &= \vec{J}_0 - \vec{X}_0 \times \vec{P}_0 \end{aligned} \tag{2}$$

The interaction $v$ may be taken as a *local* function in the relative coordinate which is conjugate to the relative momentum $p$ in the c.m. system; but since the scheme anyhow does not lead to local differential equations, there is not much to be gained from such a choice. The Wigner canonical spin $\vec{J}_0$ commutes with $\vec{P} = \vec{P}_0$ and $\vec{X} = \vec{X}_{.0}$ and is related to the Pauli-Lubanski vector $W_\mu = \varepsilon_{\mu\nu\kappa\lambda} P^\nu M^{\kappa\lambda}$ .

[9] By coupling creation channels "by hand" one can extend the DPI setting to creation/annihilation processes which still obey relativity and clustering; but thetr is no vacuum polarization and the creation through scattering has to be put in by hand instead of being a consequence of the localization principle of QFT.

As in the nonrelativistic setting, short ranged interactions $v$ lead to Møller operators and S-matrices via a converging sequence of unitaries formed from the free and interacting Hamiltonian

$$\Omega_{\pm}(H,H_0) = \lim_{t\to\pm\infty} e^{iHt}e^{-H_0t} \tag{3}$$

$$\begin{aligned}\Omega_{\pm}(M,M_0) &= \Omega_{\pm}(H,H_0) \\ S &= \Omega_+^*\Omega_-\end{aligned} \tag{4}$$

The identity in the second line is the consequence of a theorem which says that the limit is not affected if instead of $M$ one takes take a positive function of $M$ (4) as $H(M)$, as long as $H_0$ is the same function of $M_0$. This insures the the asymptotic *frame-independence of objects as the Møller operators and the S-matrix* but not that of semi asymptotic operators as formfactors of local operators between ket in and bra out particle states. Apart from this *identity for operators and their positive functions* (4) which is not needed in the nonrelativistic scattering, the rest behaves just as in nonrelativistic scattering theory. As in standard QM, the 2-particle cluster property which says that $\Omega_{\pm}^{(2)} \to \mathbf{1}$, $S^{(2)} \to \mathbf{1}$, if the two particles which interact with short range interactions are increasingly spacelike separated in the sense of the the centers of the wave packets.

The implementation of clustering is more delicate for three particles as can be seen from the fact that the first attempts were started in 1965 by Coester [22] and considerably later generalized (in collaboration with Polyzou [17]) to an arbitrary high particle number. To anticipate the result derived below, DPI leads to a consistent scheme which fulfills cluster factorization but it has no useful second quantized formulation so it may stand accused of lack of elegance; one is inclined to view less elegant theories also as less fundamental. It is also more nonlocal and nonlinear than QM, This had to be expected since adding interacting particles does not mean adding up interactions as in Schrödinger QM.

The BT form for the generators can be achieved inductively for an arbitrary number of particles. As will be seen, the advantage of this form is that in passing from n-1 to n-particles the interactions add after appropriate Poincaré transformations to the joint c.m. system and in this way one ends up with Poincaré group generators for an interacting n-particle system. But for $n > 2$ the aforementioned subtle problem with the cluster property arises; whereas this iterative construction in the nonrelativistic setting complies with cluster separability, this is not the case in the relativistic context.

This problem shows up for the first time in the presence of 3 particles [22]. The BT iteration from 2 to 3 particles gives the 3-particle mass operator

$$\begin{aligned}M &= M_0 + V_{12} + V_{13} + V_{23} + V_{123} \\ V_{12} &= M(12,3) - M_0(12;3),\ M(12,3) = \sqrt{\vec{p}_{12,3}^2 + M_{12}^2} + \sqrt{\vec{p}_{12,3}^2 + m^2}\end{aligned} \tag{5}$$

and the $M(ij,k)$ result from cyclic permutations. Here $M(12,3)$ denotes the 3-particle invariant mass in case the third particle is a "spectator", which by definition does not interact with 1 and 2. The momentum in the last line is the relative momentum between the (12)-cluster and particle 3 in the joint c.m. system and $M_{12}$ is the associated two-particle mass i.e. the invariant energy in the (12) c.m system. Written in terms of the original two-particle interaction $v$, the 3-particle mass term appears very nonlinear.

As in the nonrelativistic case, one can always add a totally connected contribution. Setting this contribution to zero, the 3-particle mass operator only depends on the two-particle interaction $v$. But contrary to the nonrelativistic case, the BT generators constructed with $M$ as it stands does not fulfill the cluster separability requirement. The latter demands that if the interaction between two clusters is removed, the unitary representation factorizes into that of the product of the two clustersn i.e. one expects that shifting the third particle to infinity will render it a spectator and result in a factorization $U_{12,3} \rightarrow U_{12} \otimes U_3$. Unfortunately what really happens is that the (12) interaction also gets switched off in this process i.e. $U_{123} \rightarrow U_1 \otimes U_2 \otimes U_3$ . The reason for this violation of the cluster separability property, as a simple calculation (using the transformation formula from c.m. variables to the original $p_i$, i = 1, 2, 3) shows [17], is that, although the spatial translation in the original system (instead of the $12,3$ c.m. system) does remove the third particle to infinity as it should, unfortunately it also drives the two-particle mass operator (with which it does not commute) towards its free value which violates clustering.

In other words the BT produces a Poincaré covariant 3-particle interaction which is additive in the respective c.m. interaction terms (5), but the Poincaré representation $U$ of the resulting system will not be cluster-separable. However this is the time for intervention of a saving grace: *scattering equivalence*.

As shown first in [22], even though the 3-particle representation of the Poincaré group arrived at by the above arguments violates clustering, the 3-particle S-matrix computed in the additive BT scheme turns out to have the cluster factorization property. But without implementing the correct cluster factorization also for the 3-particle Poincaré generators there is no chance to proceed to a clustering 4-particle S-matrix.

Fortunately there always exist unitaries which transform BT systems into cluster-separable systems *without affecting the S-matrix*. Such transformations are called *scattering equivalences.* They were first introduced into QM by Sokolov [23] and their intuitive content is related to a certain insensitivity of the scattering operator under quasilocal changes of the quantum mechanical description at finite times. This is reminiscent of the insensitivity of the S-matrix against local changes in the interpolating field-coordinatizations[10] in QFT by e.g. using composites instead of the Lagrangian field.

The notion of scattering equivalences is conveniently described in terms of

[10] In field theoretic terminology this means changing the pointlike field by passing to another (composite) field in the same equivalence class (Borchers class) or in the setting of AQFT by picking another operator from a local operator algebra.

a subalgebra of *asymptotically constant operators* $C$ defined by

$$\lim_{t\rightarrow\pm\infty} C^{\#} e^{iH_0 t}\psi = 0 \qquad (6)$$
$$\lim_{t\rightarrow\pm\infty} \left(V^{\#} - 1\right) e^{iH_0 t}\psi = 0$$

where $C^{\#}$ stands for both $C$ and $C^{*}$. These operators, which vanish on dissipating free wave packets in configuration space, form a *-subalgebra which extends naturally to a $C^{*}$-algebra $\mathcal{C}$. A scattering equivalence is a unitary member $V \in \mathcal{C}$ which is asymptotically equal to the identity (the content of the second line). Applying this asymptotic equivalence relation to the Møller operator one obtains

$$\Omega_{\pm}(VHV^{*}, VH_0V^{*}) = V\Omega_{\pm}(H, H_0) \qquad (7)$$

so that the $V$ cancels out in the S-matrix. Scattering equivalences do however change the interacting representations of the Poincaré group according to $U(\Lambda, a) \rightarrow VU(\Lambda, a)V^{*}$.

The upshot is that there exists a clustering Hamiltonian $H_{clu}$ which is unitarily related to the BT Hamiltonian $H_{BT}$ i.e. $H_{clu} = BH_{BT}B^{*}$ such that $B \in \mathcal{C}$ is determined in terms of the scattering data computed from $H_{BT}$. It is precisely this clustering of $H_{clu}$ which is needed for obtaining a clustering 4-particle S-matrix which is cluster-associated with the $S^{(3)}$. With the help of $M_{clu}$ one defines a 4-particle interaction following the additive BT prescription; the subsequent scattering formalism leads to a clustering 4-particle S-matrix and again one would not be able to go to n=5 without passing from the BT to the cluster-factorizing 4-particle Poincaré group representation. Coester and Polyzou showed [17] that this procedure can be iterated and doing this one arrives at the following statement

**Statement**: *The freedom of choosing scattering equivalences can be used to convert the Bakamijan-Thomas presentation of multi-particle Poincaré generators into a cluster-factorizing representation. In this way a cluster-factorizing S-matrix $S^{(n)}$ associated to a BT representation $H_{BT}$ (in which clustering mass operator $M_{clu}^{(n-1)}$ was used) leads via the construction of $M_{clu}^{(n)}$ to a S-matrix $S^{(n+1)}$ which clusters in terms of all the previously determined $S^{(k)}, k < n$. The use of scattering equivalences prevents the existence of a $2^{nd}$ quantized formalism.*

For a proof we refer to the original papers [17][24]. In passing we mention that the minimal extension, i.e. the one determined uniquely in terms of the two-particle interaction $v$) from n to n+1 for $n > 3$, contains *connected 3-and higher particle interactions* which are nonlinear expressions involving nested roots in terms of the original two-particle $v$. This is another unexpected phenomenon as compared to the nonrelativistic case.

This theorem shows that it is possible to construct a relativistic theory which however only uses particle concepts, thus correcting an old folklore which says relativity + clustering = QFT. Whether one should call this DPI theory

"relativistic QM" or just a relativistic S-matrix theory in a QM setting is a matter of taste; it depends on what significance one attributes to those unusual scattering equivalences. In any case it defines a *relativistic S-matrix setting* with the correct particle behavior i.e. all properties which one is able to formulate in terms of particles (without the use of fields) as unitarity, Poincaré invariance and macrocausality are fulfilled. In this context one should also mention that the S-matrix bootstrap approach never addressed these macro-causality problems of the DPI approach; it was a grand self-deluding design for a unique theory of all non-gravitational interactions in which important physical details were arrogantly ignored.

As mentioned above Coester and Polyzou also showed that this relativistic setting can be extended to processes which maintain cluster factorization in the presence of a finite number of creation/annihilation channels, thus demonstrating, as mentioned before, that *the mere presence of particle creation is not characteristic for QFT* (but rather the presence of infinite vacuum polarization clouds from "banging" with localized operators onto the vacuum, see section 7). Different from the nonrelativistic Schrödinger QM, the superselection rule for masses of particles which results from Galilei invariance for nonrelativistic QM does not carry over to the relativistic setting; in this respect DPI is less restrictive than its Galilei-invariant QM counterpart where such creation processes are forbidden.

One may consider the DPI setting of Coester and Polyzou as that scheme which results from implementing the mentioned particle properties within a n-particle Wigner representation setting in the presence of interaction [17]. Apparently the work of these mathematical nuclear physicists has not been noted by particle physicists since the authors have published most of their results in nuclear physics journals. What makes it worthwhile to mention this work is that even physicists of great renown as Steven Weinberg did not believe that such a theory exists because otherwise they would not have conjectured that the implementation of cluster factorization properties in a relativistic setting leads to QFT [25].

Certain properties which are consequences of locality in QFT and can be formulated but not derived in a particle setting as the TCP symmetry, the spin-statistics connection and the existence of anti-particles, can be added "by hand" to the DPI setting. Other properties which are on-shell relics of locality which QFT imprints on the S-matrix and which require the notion of analytic continuation in particle momenta (as e.g. the crossing property for formfactors) cannot be implemented in the QM setting of DPI.

## 3.2 QFT and modular localization

After having his feathers ruffled by Einstein on the issue of whether photons (Nadelstrahlung) are necessary for acquiring radiation equilibrium, Jordan's hopes that Einstein would support his beautiful solution in terms of field quantization in the Dreimännerarbeit, which now supported Einstein's photonsdid not materialize. He must have been somewhat disappointed with the lukewarm

reception of his calculation, which after all heralded the beginnings of QFT in the same fateful year as Heisenberg proposed his QM. Einstein's reluctance to embrace field quantization had of course nothing specifically to do with Jordan's work in the Dreimännerarbeit, but rather with his own reservation to except any form of quantum theory as the final description of the new phenomena which had the Cain's mark of probability. Even Jordan's collaborators in the Dreimännerarbeit had second thoughts about field quantization[11] for other reasons; but a critical attitude towards new speculative ideas against a young iconoclast was quite normal; that the speculative method of research at the frontier of particle theory requires a strong critical balance was quite accepted. When Schweber pictures Jordan as the unsung hero [26], he referes to the lack of recognition two decades later when most of his contemporaries received Nobel prizes except Jordan who after Einstein Pauli and Heisenberg was one of the scientific giants of the century before he left quantum physics and became involved in the turmoil of the Third Reich.

Before we comment on Jordan's view of QFT and its underlying causal locality principle, as well as his unsucessful search for an intrinsic description which does not follow the classical quantization parallellism but starts with quantum principles[12], we will remind the reader of the actual insight into this problem. In this way it is easier to to understand the distance between Jordan's world of QFT and the modern setting of an autonomous QFT.

Previously it was mentioned in passing that the localization underlying QFT can be freed from the contingencies of field coordinatizations. This is best achieved by a physically as well mathematically impressive recently discovered structure of QFT which is still little known. Its name, "modular theory" is of mathematical origin and refers to a vast generalization of the (uni)modularity encountered in the relation between left/right Haar measure in group representation theory. In the middle 60s the mathematician Tomita presented this theory as a significant addition to the theory of operator algebras and in the following years this theory received essential improvements from Takesaki and later also from Connes.

At the same time Haag, Hugenholtz and Winnink published some work on statistical mechanics of open systems [27]. When physicists and mathematicians using operator algebra methods in their research met at a conference in Baton Rouge (Luisiana, USA) in 1966, there was mutual surprise about the similarity of concepts, followed by deep appreciation of the perfection with which these independently motivated developments supported each other [28]. Physicists not only adapted mathematical concepts about operator algebras, but mathematicians also took some of their terminology from physicists as e.g. *KMS states* which refer to Kubo, Martin and Schwinger who introduced an analytic

[11] One later argument was "why should something which was already quantum be quantized a second time" is justified with respect to QM (no need to do this), but is based on a misunderstanding in the QFT context.

[12] Jordan, as other physicists with a strong philosophical background, did not accept that a less fundamental theory (classical physics) via quantization calls the shots for a more basic one.

property of thermal Gibbs states merely as a computational tool (in order to avoid computing traces), whereas Haag, Hugenholtz and Winnink realized that this property (the "KMS property") plays a foundational conceptual role; it is the only property which survives in the thermodynamic limit when the Gibbs trace formulas loses its meaning and needs to be replaced by the analytic KMS boundary condition. After the vacuum state, the KMS states are the most important states in QFT.

At that conference the relation of quantum field theoretical *localization* to the modular theory of operator algebras was still not known, although Hawking's work on thermal aspects of physics inside and outside a black hole event horizon could have already suggested that at the localization behind an event horizon generates a situation with thermal manifestations of localization are intimately related with aspects of this new modular theory. The general connection of causal localization or localization behind event horizons with KMS states and modular operator theory was made a decade after Baton Rouge and directly after Hawking's work; first in a more abstract paper in which the modular objects for wedge-localized algebras were determined[13] [30] being followed by arguments in favor of KMS thermal properties in black hole physics really originating from the application of modular theory to quantum matter enclosed behind an event horizon (the Schwarzschild horizon) [31].

The theory becomes more accessible if one introduces it first in its more limited spatial- instead of its full algebraic- context. Since as a foundational structure of LQP it merits more attention than it hitherto received in the particle physics literature, some of its methods and achievements will be presented in the sequel.

It has been realized by Brunetti, Guido and Longo[14] [33] that there exists a natural intrinsic localization structure on the Wigner representation space for any *positive energy representation* of the proper Poincaré group which is in particular independent of the Born-Newton Wigner external localization structure (which has no relation to the particular Wigner representation). The starting point is an irreducible representation $U_1$of the Poincaré´group on a Hilbert space $H_1$ that after "second quantization" becomes the single-particle subspace of the Wigner-Fock Hilbert space $H_{WF}$ on which the quantum fields act[15]. In the bosonic case the construction then proceeds according to the following steps [33][34][35].

One first fixes a reference wedge region, e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > \left|x^0\right|\}$ and considers the one-parametric L-boost group (the hyperbolic rotation by $\chi$ in the $x^{d-1} - x^0$ plane) which leaves $W_0$ invariant; one also needs the reflection $j_{W_0}$ across the edge of the wedge (i.e. along the coordinates $x^{d-1} - x^0$). The $\mathfrak{j}_{W_0}$-

[13]The abstract Bisognano-Wichmann situation can be given a more physical appearance in the form of the Unruh Gedankenexperiment [29] of a uniformely accelerated observer whose world line is a timelike hyperbola in the wedge-localized Rindler spacetime.

[14]In a more limited context and with less mathematical rigor this was independently proposed in [32].

[15]The construction works for arbitrary positive energy representations, not only for irreducible ones.

extended Wigner representation is then used to define two commuting wedge-affiliated operators

$$\delta_{W_0}^{it} = \mathfrak{u}(0, \Lambda_{W_0}(\chi = -2\pi t)),\ \mathfrak{j}_{W_0} = \mathfrak{u}(0, j_{W_0}) \tag{8}$$

where attention should be paid to the fact that in a positive energy representation any operator which inverts time is necessarily antilinear[16]. A unitary one- parametric strongly continuous subgroup as $\delta_{W_0}^{it}$ can be written in terms of a selfadjoint generator $K$ as $\delta_{W_0}^{it} = e^{-itK_{W_0}}$ and therefore permits an "analytic continuation" in $t$ to an unbounded densely defined positive operators $\delta_{W_0}^{s}$. With the help of this operator one defines the unbounded antilinear operator which has the same dense domain as its "radial" part

$$\mathfrak{s}_{W_0} = \mathfrak{j}_{W_0}\delta_{W_0}^{\frac{1}{2}},\ \mathfrak{j}\delta^{\frac{1}{2}}\mathfrak{j} = \delta^{-\frac{1}{2}} \tag{9}$$

Whereas the unitary operator $\delta_{W_0}^{it}$ commutes with the reflection, the antiunitarity of the reflection changes the sign in the analytic continuation which leads the commutation relation between $\delta$ and $\mathfrak{j}$ in (9). This causes the involutivity of the s-operator on its domain, as well as the identity of its range with its domain

$$\mathfrak{s}_{W_0}^2 \subset \mathbf{1}$$
$$dom\ \mathfrak{s} = ran\ \mathfrak{s}$$

Such operators which *are unbounded and yet involutive* on their domain are very unusual; according to my best knowledge they only appear in modular theory and it is precisely these unusual properties which are capable to encode geometric localization properties into domain properties of abstract quantum operators, a fantastic achievement completely unknown in QM. The more general algebraic context in which Tomita discovered modular theory will be mentioned later.

The involutivity means that the s-operator has $\pm 1$ eigenspaces; since it is antilinear, the +space multiplied with $i$ changes the sign and becomes the -space; hence it suffices to introduce a notation for just one eigenspace

$$\begin{aligned} \mathfrak{K}(W_0) &= \{domain\ of\ \Delta_{W_0}^{\frac{1}{2}},\ \mathfrak{s}_{W_0}\psi = \psi\} \qquad (10)\\ \mathfrak{j}_{W_0}\mathfrak{K}(W_0) &= \mathfrak{K}(W_0') = \mathfrak{K}(W_0)',\ duality \\ \overline{\mathfrak{K}(W_0) + i\mathfrak{K}(W_0)} &= H_1,\ \mathfrak{K}(W_0) \cap i\mathfrak{K}(W_0) = 0 \end{aligned}$$

It is important to be aware that, unlike QM, we are here dealing with real (closed) subspaces $\mathfrak{K}$ of the complex one-particle Wigner representation space $H_1$. An alternative which avoids the use of real subspaces is to directly work with complex dense subspaces as in the third line. Introducing the graph norm of the dense space the complex subspace in the third line becomes a Hilbert space in its own right. The second and third line require some explanation.

[16]The wedge reflection $\mathfrak{j}_{W_0}$ differs from the TCP operator only by a $\pi$-rotation around the $W_0$ axis.

The upper dash on regions denotes the causal disjoint (which is the opposite wedge) whereas the dash on real subspaces means the symplectic complement with respect to the symplectic form $Im(\cdot,\cdot)$ on $H_1$.

The two properties in the third line are the defining property of what is called the *standardness property* of a real subspace[17]; any standard K space permits to define an abstract s-operator

$$\begin{aligned}\mathfrak{s}(\psi+i\varphi)&=\psi-i\varphi \\ \mathfrak{s}&=\mathfrak{j}\delta^{\frac{1}{2}}\end{aligned} \tag{11}$$

whose polar decomposition (written in the second line) yields two modular objects, a unitary modular group $\delta^{it}$ and a antiunitary reflection which generally have however no direct geometric significance. The domain of the Tomita $\mathfrak{s}$-operator is the same as the domain of $\delta^{\frac{1}{2}}$ namely the real sum of the K space and its imaginary multiple. Note that this domain is determined solely in terms of Wigner group representation theory.

It is easy to obtain a net of K-spaces by $U(a,\Lambda)$-transforming the K-space for the distinguished $W_0$. A bit more tricky is the construction of sharper localized subspaces via intersections

$$\mathfrak{K}(\mathcal{O})=\bigcap_{W\supset\mathcal{O}}\mathfrak{K}(W) \tag{12}$$

where $\mathcal{O}$ denotes a causally complete smaller region (noncompact spacelike cone, compact double cone). Intersection may not be standard, in fact they may be zero in which case the theory allows localization in $W$ (it always does) but not in $\mathcal{O}$. Such a theory is still causal but not local in the sense that its generating free fields are pointlike. One can show that the noncompact intersection for spacelike cones $\mathcal{O}=\mathcal{C}$ for all positive energy is always standard.

Note that the relativistic DPI setting also starts from Wigner particles, but it ignores the presence of this autonomous and covariant modular localization structure and instead introduces quantum mechanical frame-dependent Born-Newton-Wigner localization based on position operators which have no intrinsic relation to the Wigner representation theory of the Poincaré group. It is easy to show that the dense subspaces $\mathfrak{K}(\mathcal{O})+i\mathfrak{K}(\mathcal{O})$ which are the domain of the unbounded Tomita involution $\mathfrak{s}$ have all the properties demanded of a net of $\mathcal{O}$-localized subspaces of the one-particle Wigner space. The subtlety of causal localization here is, that whereas BNW localized subspaces are (as spectral subspaces of a selfadjoint position operator) closed subspaces, the frame-independent modular subspaces are dense and change with the localization region; in fact the spacetime localization is fully encoded in the dense domain of the modular $\mathfrak{s}(\mathcal{O})$ operator.

[17] According to the Reeh-Schlieder theorem [27] a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

There are three classes of irreducible positive energy representation, the family of massive representations $(m > 0, s)$ with half-integer spin $s$ and the family of massless representation which consists really of two subfamilies with quite different properties, namely the $(0, h =$ half-integer$)$ class, often called the neutrino-photon class, and the rather large class of $(0, \kappa > 0)$ infinite helicity representations parametrized by a continuous-valued Casimir invariant $\kappa$ [35].

For the first two classes the $\mathfrak{K}$-space the standardness property also holds for double cone intersections $\mathcal{O} = \mathcal{D}$ for arbitrarily small $\mathcal{D}$, but this is definitely not the case for the infinite helicity family for which the localization spaces for compact spacetime regions turn out to be trivial[18]. Passing from localized subspaces $\mathfrak{K}$ in the representation theoretical setting to singular covariant generating wave functions (the "first quantized" analogs of generating fields) one can show that the $\mathcal{D}$ localization leads to pointlike singular generators (state-valued distributions) whereas the spacelike cone localization $\mathcal{C}$ is associated with semi-infinite spacelike stringlike singular generators [35]. Their "second quantized" counterparts are pointlike or (in case of the infinite spin family) stringlike covariant fields. It is remarkable that the modular localization concept does not require to introduce generators which are localized on timelike hypersurfaces (branes).

Although the observation that the third Wigner representation class is not pointlike generated was made many decades ago, the statement that it is semi-infinite string-generated and that this is the worst possible case of state localization is based on modular theory and is of a more recent vintage [33][35].

There is a very subtle aspect of modular localization which one encounters in the second Wigner representation class of massless finite helicity representations (the photon, graviton class). Whereas in the massive case all spinorial fields $\Psi^{(A,\dot{B})}$ the relation of the physical spin $s$ with the two spinorial indices follows the naive angular momentum composition rules [25]

$$\begin{aligned} \left|A - \dot{B}\right| \leq s \leq \left|A + \dot{B}\right|,\ m > 0 \\ s = \left|A - \dot{B}\right|,\ m = 0 \end{aligned} \tag{13}$$

the second line contains the significantly reduced number of spinorial descriptions for zero mass and finite helicity representations. What is going on here, why is there, in contradistinction to classical field theory, no covariant s=1 vector-potential $A_\mu$ or no $g_{\mu\nu}$ in case of s=2 ? Why are the admissible covariant generators of the Wigner representation in this case limited to field strengths (for s=2, the linearized Riemann tensor) and why do their potentials inherit this localization only in the massive but not in the massless case ?

The short answer is that there is really a deep clash between the quantum aspects (Hilbert space structure) and spacetime localization which in the

[18] It is quite easy to prove the standardness for spacelike cone localization (leading to singular stringlike generating fields) just from the positive energy property which is shared by all three families [33].

$(m=0, s\geq 1)$ family (different from the infinite spin family) does not affect the representation as such but rather the existence of certain generators with prescribed covariance properties (those not appearing in the second line of (13)). In the gauge theory formulation one sacrifices the Hilbert space and maintains (at least formally[19]) a pointlike localization in an indefinite metric Krein space whereas maintaining the Hilbert space means relaxing the covariant localization from point to semiinfinite stringlike. But this is not a bad compromise, the physical localization is really stringlike generated. Taking this option, the full range of spinorial possibilities (13) returns in terms of string localized fields $\Psi^{(A,\dot{B})}(x,e)$ if $s\neq \left|A-\dot{B}\right|$. These generating free fields are covariant and "string-local"

$$\begin{aligned} U(\Lambda)\Psi^{(A,\dot{B})}(x,e)U^{*}(\Lambda) &= D^{(A,\dot{B})}(\Lambda^{-1})\Psi^{(A,\dot{B})}(\Lambda x,\Lambda e) \\ \left[\Psi^{(A,\dot{B})}(x,e),\Psi^{(A^{\prime},\dot{B}^{\prime})}(x^{\prime},e^{\prime})\right]_{\pm} &= 0,\ x+\mathbb{R}_{+}e >< x^{\prime}+\mathbb{R}_{+}e^{\prime} \end{aligned} \tag{14}$$

Here the unit vector $e$ is the spacelike direction of the semiinfinite string and the last line expresses the spacelike fermionic/bosonic spacelike commutation. The best known illustration is the $(m=0, s=1)$ vectorpotential representation; in this case it is well-known that although a generating pointlike field strength exists, there is no *pointlike* vectorpotential acting in a Hilbert space.

According to (14) the modular localization approach offers as a substitute a stringlike covariant vector potential $A_{\mu}(x,e)$. In the case $(m=0, s=2)$ the "field strength" is a fourth degree tensor which has the symmetry properties of the Riemann tensor (it is often referred to as the *linearized* Riemann tensor). In this case the string-localized potential is of the form $g_{\mu\nu}(x,e)$ i.e. resembles the metric tensor of general relativity. Some consequences of this localization for a reformulation of gauge theory will be mentioned in section 8.

Even in case of massive free theories where the representation theoretical approach of Wigner does not require to go beyond pointlike localization, covariant stringlike localized fields exist. Their attractive property is that they improve the short distance behavior e.g. a massive pointlike vector-potential of *sdd=2* passes to a string localized vector potential of *sdd=1.* In this way the increase of the sdd of pointlike fields with spin s can be traded against string localized fields of spin independent dimension with sdd=1. This observation would suggest the possibility of an enormous potential enlargement of perturbative accessible higher spin interaction in the sense of power counting.

A different kind of spacelike string-localization arises in d=1+2 Wigner representations with anomalous spin [36]. The amazing power of the modular localization approach is that it preempts the spin-statistics connection already in the one-particle setting, namely if s is the spin of the particle (which in d=1+2 may take on any real value) then one finds for the connection of the symplectic

[19]From a physical viewpoint the retention of the pointlike nature is a Pyrrhic victory since this localization is void of physical meaning.

complement with the causal complement the generalized duality relation

$$\mathfrak{K}(\mathcal{O}') = Z\mathfrak{K}(\mathcal{O})' \tag{15}$$

where the square of the twist operator $Z = e^{\pi is}$ is easily seen (by the connection of Wigner representation theory with the two-point function) to lead to the statistics phase $= Z^2$ [36].

The fact that one never has to go beyond string localization (and fact, apart from $s \geq 1$, never beyond point localization) in order to obtain generating fields for a QFT is remarkable in view of the many attempts to introduce extended objects into QFT.

It is helpful to be again reminded that modular localization which goes with real subspaces (or dense complex subspaces), unlike B-N-W localization, cannot be connected with probabilities and projectors. It is rather related to causal localization aspects; the standardness of the K-space for a compact region is nothing else then the one-particle version of the Reeh-Schlieder property. As will be seen in the next section modular localization is also an important tool in the non-perturbative construction of interacting models.

A net of real subspaces $\mathfrak{K}(\mathcal{O}) \subset H_1$ for an finite spin (helicity) Wigner representation can be "second quantized"[20] via the CCR (Weyl) respectively CAR quantization functor; in this way one obtains a covariant $\mathcal{O}$-indexed net of von Neumann algebras $\mathcal{A}(\mathcal{O})$ acting on the bosonic or fermionic Fock space $H = Fock(H_1)$ built over the one-particle Wigner space $H_1$. For integer spin/helicity values the modular localization in Wigner space implies the identification of the symplectic complement with the geometric complement in the sense of relativistic causality, i.e. $\mathfrak{K}(\mathcal{O})' = \mathfrak{K}(\mathcal{O}')$ (spatial Haag duality in $H_1$). The Weyl functor takes this spatial version of Haag duality into its algebraic counterpart. One proceeds as follows: for each Wigner wave function $\varphi \in H_1$ the associated (unitary) Weyl operator is defined as

$$\begin{aligned} Weyl(\varphi) &:= expi\{a^*(\varphi) + a(\varphi)\} \in B(H) \\ \mathcal{A}(\mathcal{O}) &:= alg\{Weyl(\varphi)|\varphi \in \mathfrak{K}(\mathcal{O})\}'', \;\; \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \end{aligned} \tag{16}$$

where $a^*(\varphi)$ and $a(\varphi)$ are the usual Fock space creation and annihilation operators of a Wigner particle in the wave function $\varphi$. We then define the von Neumann algebra corresponding to the localization region $\mathcal{O}$ in terms of the operator algebra generated by the functorial image of $\mathfrak{K}(\mathcal{O})$ as in the second line. By the von Neumann double commutant theorem, our generated operator algebra is weakly closed.

The functorial relation between real subspaces and von Neumann algebras via the Weyl functor preserves the causal localization structure and hence the spatial duality passes to its algebraic counterpart. The functor also commutes

[20] The terminology $2^{nd}$ quantization is a misdemeanor since one is dealing with a rigorously defined functor within QT which has little in common with the artful use of that parallellism to classical theory called "quantization". In Edward Nelson's words: (first) quantization is a mystery, but second quantization is a functor.

with the improvement of localization through intersections $\cap$ according to $\mathfrak{K}(\mathcal{O}) = \cap_{W\supset O}\mathfrak{K}(W),\ \mathcal{A}(\mathcal{O}) = \cap_{W\supset O}\mathcal{A}(W)$ as expressed in the commuting diagram

$$\begin{array}{ccc} \{\mathfrak{K}(W)\}_W & \longrightarrow & \{\mathcal{A}(W)\}_W \\ \downarrow\cap & & \downarrow\cap \\ \mathfrak{K}(\mathcal{O}) & \longrightarrow & \mathcal{A}(\mathcal{O}) \end{array} \tag{17}$$

Here the vertical arrows denote the tightening of localization by intersection whereas the horizontal ones indicate the action of the Weyl functor.

The case of half-integer spin representations is analogous [34], apart from the fact that there is a mismatch between the causal and symplectic complements which must be taken care of by a *twist operator* $\mathcal{Z}$ and as a result one has to use the CAR functor instead of the Weyl functor.

In case of the large family of irreducible zero mass infinite spin representations in which the lightlike little group is faithfully represented, the finitely localized K-spaces are trivial $\mathfrak{K}(\mathcal{O}) = \{0\}$ and the *most tightly localized nontrivial spaces are of the form* $\mathfrak{K}(\mathcal{C})$ for $\mathcal{C}$ an arbitrarily narrow *spacelike cone.* As a double cone contracts to its core which is a point, the core of a spacelike cone is a *covariant spacelike semiinfinite string.* The above functorial construction works the same way for the Wigner infinite spin representation, except that in that case there are no nontrivial algebras which have a smaller localization than $\mathcal{A}(\mathcal{C})$ and there is no field which is sharper localized than a semiinfinite string. As stated before, stringlike generators, which are also available in the pointlike case, turn out to have an improved short distance behavior which makes them preferable from the point of view of formulating interactions within the power counting limit. They can be constructed from the unique Wigner representation by so called intertwiners between the unique canonical and the many possible covariant (dotted-undotted spinorial) representations. The Euler-Lagrange aspects plays no direct role in these construction since the causal aspect of hyperbolic differential propagation are fully taken care of by modular localization and also because most of the spinorial higher spin representations (13) cannot be characterized in terms of Euler-Lagrange equations. Modular localization is the more general method of implementating causal propagation than that in terms of hyperbolic equations of motions.

A basis of local covariant field coordinatizations is then defined by the free field and its Wick-ordered composites. The case which deviates furthest from classical behavior is the pure stringlike infinite spin case which relates a *continuous* family of free fields with one irreducible infinite spin representation. Its non-classical aspects, in particular the absence of a Lagrangian, is the reason why the spacetime description in terms of semiinfinite string fields has been discovered only recently rather than at the time of Jordan's field quantization or Wigner's representation theoretical approach.

Using the standard notation $\Gamma$ for the second quantization functor which maps real localized (one-particle) subspaces into localized von Neumann algebras and extending this functor in a natural way to include the images of the $\mathfrak{K}(\mathcal{O})$-associated $s, \delta, j$ which are denoted by $S, \Delta, J$, one arrives at the Tomita

Takesaki theory of the interaction-free local algebra $(\mathcal{A}(\mathcal{O}),\Omega)$ in standard position[21]

$$\begin{aligned} &H_{Fock}=\Gamma(H_1)=e^{H_1},\ \left(e^h,e^k\right)=e^{(h,k)} \\ &\Delta=\Gamma(\delta),\ J=\Gamma(j),\ S=\Gamma(s) \\ &SA\Omega=A^*\Omega,\ A\in\mathcal{A}(O),\ S=J\Delta^{\frac{1}{2}} \end{aligned} \tag{18}$$

This result is a special case of the Tomita-Takesaki theorem which is a statement about the existence of two modular objects $\Delta^{it}$ and $J$ on the algebra

$$\begin{aligned} \sigma_t(\mathcal{A}(\mathcal{O}))\equiv\Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it}=\mathcal{A}(\mathcal{O}) \\ J\mathcal{A}(\mathcal{O})J=\mathcal{A}(\mathcal{O})'=\mathcal{A}(\mathcal{O}') \end{aligned} \tag{19}$$

in words: the reflection $J$ maps an algebra (in standard position) into its von Neumann commutant and the unitary group $\Delta^{it}$ defines an one-parametric automorphism-group $\sigma_t$ of the algebra. In this form (but without the last geometric statement involving the geometrical causal complement $\mathcal{O}'$) the theorem hold in complete mathematical generality for standard pairs $(\mathcal{A},\Omega)$. The free fields and their Wick composites are "coordinatizing" singular generators of this $\mathcal{O}$-indexed net of operator algebras in that the smeared fields $A(f)$ with $suppf\subset\mathcal{O}$ are (unbounded operators) affiliated with $\mathcal{A}(\mathcal{O})$ and in a certain sense generate $\mathcal{A}(\mathcal{O})$.

In the above second quantization context the origin of the T-T theorem and its proof is clear: the symplectic disjoint passes via the functorial operation to the operator algebra commutant (13) and the spatial one-particle automorphism goes into its algebraic counterpart. The definition of the Tomita involution $S$ through its action on the dense set of states (guarantied by the standardness of $\mathcal{A}$) as $SA\Omega=A^*\Omega$ and the action of the two modular objects $\Delta,J$ (18) is however part of the general setting of the modular Tomita-Takesaki theory of abstract operator algebras in "standard position"; standardness is the mathematical terminology for the physicists Reeh-Schlieder property i.e. the existence[22] of a vector $\Omega\in H$ with respect to which the algebra acts cyclic and has no "annihilators" of $\Omega$. Naturally the proof of the abstract T-T theorem in the general setting of operator algebras is more involved[23]. It validity can be established in interacting QFT either in a Wightman [38] setting or in theories which have a complete scattering interpretation [39]

The domain of the unbounded Tomita involution $S$ turns out to be "kinematical" in the sense that the dense set which features in the Reeh-Schlieder theorem is determined in terms of the representation of the connected part of

[21]The functor $\Gamma$ preserves the standardness i.e. maps the spatial one-particle standardness into its algebraic counterpart.

[22]In QFT any finite energy vector (which of course includes the vacuum) has this property as well as any nondegenerated KMS state. In the mathematical setting it is shown that standard vectors are "$\delta-$dense" in $H$.

[23]The local algebras of QFT are (as a consequence of the split property) hyperfinite; for such operator algebras Longo has given an elegant proof [37].

the Poincaré group i.e. the particle/spin spectrum[24]. In other words the Reeh-Schlieder domains in an interacting theory with asymptotic completeness are identical to those of the incoming or outgoing free field theory.

The important property which renders this useful beyond free fields as a new constructive tool in the presence of interactions, is that for $(\mathcal{A}(W), \Omega)$ the antiunitary involution $J$ depends on the interaction, whereas $\Delta^{it}$ continues to be uniquely fixed by the representation of the Poincaré group i.e. by the particle content. In fact it has been known for some [32] time that $J$ is related with its free counterpart $J_0$ through the scattering matrix

$$J = J_0 S_{scat} \tag{20}$$

This modular role of the scattering matrix as a relative modular invariant between an interacting theory and its free counterpart comes as a surprise. It is precisely this property which opens the way for an inverse scattering construction. Hence the properties of $J$ are essentially determined by the relation of localized operators $A$ to their Hermitian adjoints $A^*$[25].

Jordan had an intuitive understanding of causal localization in analogy to the causal propagation (Cauchy propagation) in classical field theories as e.g. the Maxwell theory. But his observation that in gauge theories the gauge invariant physical matter fields were not point- but string- localized (section 6) was not pursued to its foundational roots.

The string localization of potentials plays an essential role in understanding the origin the nonlocality of charged and colored quantum matter as a result of its interaction with nonlocal potentials. In this way the Dirac-Jordan-Mandelstam "string" ( see section6 (32)) is replaced by the matter field in a new perturbation theory in which "gauge" has been replaced by noncompact stringlike localization in a Hilbert space (no indefinite metric) setting such that the compact localized pointlike generated subalgebra coincides with the gauge invariant subalgebra of the standard gauge approach [40].

An important mechanism which has been almost forgotten in the present discourse about the expectations from the impending LHC experiments is the Schwinger-Higgs screening mechanism. The idea is that gauge theory with scalar matter has a screened counterpart. This setting has conceptual differences with the Higgs mechanism. There is a clear distinction to the Goldstone spontaneous symmetry breaking; whereas for the latter the conserved current leads to an infinite charge (large distance divergence from presence of Goldstone Boson), the screening mechanism leads to a vanishing charge and conversion of complex matter into real matter. The ensuing loss of the charge superselection rule

[24]For a wedge $W$ the domain of $S_W$ is determined in terms of the domain of the "analytic continuation" $\Delta_W^{\frac{1}{2}}$ of the wedge-associated Lorentz-boost subgroup $\Lambda_W(\chi)$, and for subwedge localization regions $\mathcal{O}$ the dense domain is obtained in terms of intersections of wedge domains.

[25]According to a theorem of Alain Connes [41] the existence of operator algebras in standard position can be inferred if the real subspace $K$ permit a decompositions into a natural positive cone and its opposite with certain facial properties of positive subcones. Although this construction has been highly useful in Connes classification of von Neumann factors, it has not yet been possible to relate this to physical concepts.

leads to a loss of symmetry which affects even the even/odd symmetry of the remaining matter field [40].

## 3.3 Speculative ideas without critical counter-balance

QFT was born almost simultaneous with QM in a critical surrounding. Not only did it have to assert itself against older proposals based on Bohr's model but without photons, but even Einstein's support was muffled, and Jordan's coauthors Born and Heisenberg maintained a critical detachment, at least for some time. Dirac was one of the first who embraced QM and became the leading figure in its shaping, but he needed almost 2 decades to wholeheartedly embrace QFT as a unifying principle in relativistic particle theory. The first pre-renormalization textbooks by Wenzel and Heitler used the QFT terminology, but employed in large parts the setting of relativistic QM which the lowest so-called tree approximation is nearly indistinguishable from QFT. The litmus test for QFT versus relativistic QM came with the problem of how to treat interaction-induced vacuum polarization i.e. how to cope with renormalization theory (higher order charged loops) which is the heart piece of the perturbative approach to QFT and the property which sharply separates QFT from relativistic QM.

In the later part of his life Jordan's research was limited to classical general relativity as well as pure mathematical problems of algebraic structures. In this way the postwar development of renormalized perturbation theory of QED as well as the subsequent derivation of dispersion relations in particle theory (which vindicated the causality ideas of Jordan for the first time in the laboratory) took place outside his attention. Even though Jordan was the first to transfer the ideas of classical gauge theory to the realm of quantum physics, he did not take notice of the renaissance of gauge theory culminating in the standard model.

A less balanced train of thoughts started with S-matrix theory. The S-matrix is, whenever it exists, a global (nonlocal) object which can be computed from the fields, but in contrast to the letter it is free of vacuum polarization and therefore also free of ultraviolet divergences. Another reason came into the foreground after renormalized perturbation theory took some of the menace out of the ultraviolet divergence issue. It was the failure of perturbative QFT to account for the strong nuclear interactions; as a result the attention shifted to more phenomenological methods closer to experimental accessible quantities as scattering cross sections. If it comes to nuclear interactions hardly anybody would claim that a nucleon or meson field is a measurable quantity of direct physical significance.

The emphasis on the S-matrix forced particle theoreticians to think somewhat harder about the field-particle relation with the result that, although particle states are important states in the Hilbert space[26], which admit an asymptotic relation to fields through the LSZ scattering theory, there is no direct field-particle relation in finite localization regions. QFT demystified the

[26]In the most important case of asymptotic completeness the Hilbert space even has the structure of a Wigner-Fock space multiparticle space.

*particle-wave duality* (e.g. the Schrödinger *wave* functions describing *states of particles*), but although the quite different *particle-field relation* is much better understood than at the time of Jordan, it did not yet arrive at its closure and it may well turn out that the complete understanding of this relation is identical to the closure of QFT. But as a rule of thumb in interacting QFT: *the larger the spacetime localization region $O$, the easier it is to control the vacuum polarization admixtures* to particle states obtained from applying operators $A(\mathcal{O})$ to the vacuum $\Omega$.

The first application of these results to the mentioned particle-adapted Kramers-Kronig dispersion theory was a resounding success. In fact it was a project of considerable foundational value in that the causal locality concept (on which QFT, in contradistinction to QM, is based on) was put to a test up to the at that time highest available energies way beyond those obtained in QED experiments one decade before. This project was well-defined and limited in scope. In contradistinction to other high energy experiments this was not a check on a particular model, but rather on a principle which was shared by all models of QFT but not by QM. Without a rigorous mathematical-conceptual derivation of these relations from the causality and spectral principles of QFT, the subsequent experimental verification would not have had the enormous significance for the continued credibility in the causal locality principles of QFT lasting into the present. This research required an S-matrix setting with the dispersion theoretic analytic properties (the relation between the real and the absorptive part of the two-particle scattering amplitude) coming from the causality principle of QFT. This should be distinguished from a pure S-matrix approach.

The project attracted the attention of the leading physicists in the 50s and 60s and it was successfully completed after less than a decade; in fact it is the only project ever in particle theory which ended with a "mission accomplished" seal. All other projects are either ongoing at a very slow pace (the standard model research) or entangled themselves in conceptual contradiction as the post dispersion relation S-matrix setting starting with the S-matrix bootstrap, passing to the dual model which in turn led into the so-called string theory (no relation to the QFT strings of the previous section!) which together with supersymmetry dominated the thinking of many particle theorists. These developments never arrived at a theoretically consistent setting nor did any important observational accessible prediction arise from the almost 50 years dominance of these ideas.

In order to be not misunderstood, a foundational theory should not be subjected to a time limit and neither should it be terminated for its lack of observational success. After all, what is at stake according to the followers of string theory, is a third kind of relativistic QT on the side of Jordan's QFT and Dirac's relativistic QM (or DPI), a theory which is claimed to be string-localized and rather unique and incorporates gravity (a "theory of everything") and extends QFT. In the good times of particle theory these claims would have been more credible because there would have been people like Oppenheimer, Pauli, Feynman, Schwinger, Lehmann, Jost, Källén...who would have looked at them in a very critical manner. What really caused considerable damage to particle

physics is that the critical balance broke down and only historians may find out whether this was caused by the ideological zeal of the string community or the ivory tower mentality of those who could have known better.

Even if nature would not make use of such a theory, the conceptual-philosophical implication of its mere existence would be startling and certainly reveal new aspects of QFT. Although QFT can be characterized as the theory which realizes the principle of modular localization, there are many aspects between the principles and the properties of models which we do not know yet; contrary to the claim of some string theorists, their opinion that QFT has reached its closure and is superseeded by string theory is an illusion resulting from the belief that it is limited to what one finds about it in textbooks.

Before we look at the nature and the causes for the conceptual derailments of the S-matrix based dual model and of string theory in particular, it is important to emphasize that this has nothing to do with a decrease of the intellectual capacity as compared to authors working in QFT at the time of Jordan, it is rather related to the loss of the delicate equilibrium between speculative exploration at the frontiers of research and its critical reflection. In a globalized community whose main purpose is to do research on a scientific monoculture and strengthen its hegemony, there is no place for a culture of autocriticism. Whereas an individual researcher would react to critique with attention and sensitivity, somebody embedded in such a community typically acts according to the vernacular that "that many cannot err". For people who think that I am exaggerating, my recommendation is to read what a critical insiders has to say about the situation [42].

In the following I will analyze two closely interconnected claims which dominated relativistic QT for many decades. They address the central issue of relativistic localization which from the time of Jordan to the present has grown in relevance and is now *the* defining property of QFT (previous subsection). Both theories, namely string theory and its predecessor the dual model, contain incurable conceptual errors resulting from a misunderstanding of localization and the consequences of stringlike localization on the S-matrix. Since the lessons to be learned from the exposition of error on a subtle problem are often not less important as those from a valid theory, the remainder of this subsection will contain detailed conceptual-mathematical arguments why the following two claims which undermine the ideas around the dual model and string theory are valid.

1) String theory as obtained by canonical quantization of the Nambu-Goto Lagrangian does not deal with string-localized objects in spacetime but rather a free string describes a "dynamically infinite component pointlike field"

2) A multicomponent chiral conformal theory has no string-like embedding into its "target space" i.e. its inner symmetry index space even in those cases in which the inner symmetry group of the chiral theory can be interpreted as a space on which the Poincaré group acts.

The first wrong track concerning the localization of the quantum theory of

the canonical quantized bilinearized[27] Nambu-Goto Lagrangian [44] came from the analogy

$$\mathcal{L}_{part} = -mc\sqrt{ds^2} \curvearrowright \; relativistic\ particle \tag{21}$$
$$\mathcal{L}_{string} = -\frac{1}{2\pi\alpha}\sqrt{dArea} \curvearrowright \; relativistic\ string$$

An incorrect idea already entered into the first line. Although metaphoric pictures are sometimes helpful, this one is patently wrong. There are no covariant position operators and a fortiori no quantum mechanical covariant world line; even in a so-called relativistic QM (DPI) it is only the M*o*ller operator and the S-matrix which are invariant, but there are no Lorentz covariant (spinorial) operators. The first who realized this was Jordan's contemporary Eugen Wigner. In fact the correct description for relativistic particles is in terms of Wigner's representation theoretical classification, whereas the above metaphoric description was only invented as a confidence inspiring metaphor by string theorist. Needless to add that the mathematical problems related to the "reparametrization invariance" of such a square root action create infinities whose removal remains part of metaphoric rather than of mathematical physics. Subscribing to such a picture of particles does not only mean abandoning Wigner's crystal clear particle classification theory, but it also leads to a conflict with the notion of relativistic localization. This is certainly not a helpful analogy to string theory.

It is perfectly possible to calculate the wave function space and the associated free "string" without relying in such doubtful metaphors. This was first done by string theorists [45][46] and the result was that the (graded in the supersymmetric case) *commutator is that of an infinite component pointlike-localized field*. Unfortunately the community pressure had already created a scissor in the head of these authors so that they simply declared the point of localization as lying on a (presumably invisible) string. Who would have expected an affirmative conclusion from a correct calculation against the prevailing view of their string community was left disappointed. A further study of the sociological mechanisms which operate in a situation where even a correct calculation serves to support incorrect ideas, should be left to future historians and philosophers of science. Errors occur wherever humans work on innovative problems and can even be found in Jordan's oeuvre, however nowadays the protective community aspects adds a different dimension by globalizing them.

The incorrect localization assignment did not start with ST; in a slightly different form it was already present in the dual model. This is obvious in the operator representations of this model, which consists in expressing the would be scattering amplitudes in terms of operators from a multi-component chiral conformal theory, in which the chiral fields are formally exponential of poten-

[27] The original Lagragian in terms of the square root of the surface element is an integrable system unrelated to string theory as we know it [43]. Its classical solutions can be related to a linear equation of motion structure which in turn may be encoded into a bilinear Lagrangian which is susceptible to canonical quantization.

tials $\Phi_k$ of an n-component abelian current model[28]. The dual model reading interprets the anomalous scale dimensions of these objects as masses of particles and the numerical vector $\vec{\alpha}$ in the exponential $exp\sum i\vec{\alpha}\vec{\Phi}$ as a particle momentum whereas the chiral field $\Phi_k(\tau)$ k=1...$n$ is assigned the role of a position $X_k(\tau)$ operator in an n-dimensional space which traces out a string as $\tau$ runs around a circle (taking the compact description of chiral conformal QFT). With other words this object is thought of as defining an embedding of a string into the n-dimensional component space called "the target space". Apart from a misunderstanding of field theoretic localization in an imagined setting of relativistic position operators, there is of course also the extremely strange idea of embedding a QFT into what is normally called its internal symmetry space.

To make a long story short, it is possible to find a representation of the Poincaré group in the internal symmetry space of a chiral theory and if one requires this representation to be unitary and of positive energy there is essentially only one solution: the so called n=10 superstring (which requires some of the $\vec{\Phi}$ components to be spinorial instead of vectorial). But the result describes (as in the previous case of the Nambu-Goto Lagrangian) a pointlike object; the stringlike nature was, as in the previous case, just a metaphoric conjecture. In fact the localization concept of QFT simply does not permit an embedding of a lower into a higher dimensional theory; the aforementioned stringlike fields do not result from an imbedding of a one dimensional chiral theory into a higher dimensional space.

In this respect localization in QFT is completely different from that in the better known QM. Whereas in QM one can simply take a one-dimensional string and promote it to a string in higher dimensions by adding coordinates, this is not possible since the causal localization is a holistic concept which is already evident from the fact that the restriction of the vacuum state to the observables localized in a spacetime regions is a highly subtle change in that on the smaller collection of observables in that spacetime region leads to the vacuum loosing its purity property and becoming a singular (no density matrix description) KMS thermal state[29]. In fact its holistic nature is the reason why geometry as a mathematical discipline and the holistic geometry based on modular quantum localization remain two different pairs of shoes. The first is neutral with respect to applications[30] whereas the is QFT context-dependent. This also explains why attempts to let QFT dance according to the tune of geometric mathematical ideas does not really clarify physical aspects.

An interesting historical illustration of the weakness of a mere geometric

[28]The exponential model has therefore the form of an n-component conformal nonlinear sigma model on which the n-component Poincaré acts in the form of an inner symmetry [47] .

[29]The KMS Hamiltonian is only physical if the localization comes from an objective event horizons of black holes rather than being an observer.dependent "Gedanken-localization" in Minkowski spacetime.

[30]For example the theory of Riemann surfaces has applications in 3-dim. geometry, the theory of analytic functions, theory of Fuchsian groups etc. Saying however that a chiral model of QFT lives on a Riemann surface is misleading since its physical localization is on one-dimensional submanifolds.

point of view is the Wess-Zumino-Witten-Novikov action. Before it acquired this long name, these models had been dealt with representation theoretical methods of current algebras in the setting of a "multicomponent Thirring model". At this level an association with a Lagrangian and a euclidean action was unclear and also not needed since a representation theoretical approach is intrinsic and self-sufficient. The only thing which a Lagrangian may add is compact elegant presentation by reading it back into classical field theory. Although Witten gave a positive answer [48] (and Novikov added mathematical insight) to this question in the form of a conformal sigma-model field, the occurrence of a topological 3-dimensional euclidean boundary term whose influence on spacetime localization remained obscure and separates this topological Lagrangian from standard Lagrangians which are susceptible to perturbative approximations. Fact is that the only constructive results on two-dimensional conformal [49] or massive non-trivial models [50] did not come from classically inspired geometric Lagrangian or functional ideas but rather from algebraic representation-theoretic methods based on modular localization.

On the other hand there is no other formalism in QT which connects so seamless to the old quasiclassical Bohr-Sommerfeld QT and perturbation theory as the Lagrangian- and functional integral. setting; the transition to the full QM would have been a home game in the setting of Feynman's pathe integral without going through all unaccustomed new nongeometric concepts in the work of Heisenberg and the Dreimännerarbeit. But perhaps we would not have been that happy with such a course of events, because although to reproduce quasiclassical results and arrive at some perturbative computations would have been simple, the exact calculation of integrable systems as the hydrogen atom would have remained inaccessible, not to speak about the important concepts as operators, states and the Born probabilty which the path integral does not deliver by itself and which only with sufficient hindsight can be extracted.

The situation becomes worse in QFT where such representations also have an intuitive metaphoric appeal[31] but no mathematical status[32]. As euclidean representations they are far removed from modular localization properties of section 3.2, hence it is not surprising that already in QM they they are inferior to operator methods, less so in QFT where modular operator algebra methods recently led to the first existence proofs for factorizing d=1+1 models [50] (which maybe viewed as the simplest field theoretical analogs of intrgrable QM). Apart from superrenormalizable models in d=1+1 Lagrangian and functional methods have not attained a mathematical status outside QM. As a result of their intuitive appeal, they are playing the role of the great communicator between particle theorists with different theoretical background. The writing down

[31] One obtains the correct perturbative combinatoric and with a bit of hindsight about what renormalization means one can calculate correlation functions. However no matter what one does, the renormalized correlations do not obey the functional integral representation i.e. the construction is a one-way street.

[32] Any method, even if it is entirely metaphoric, is useful as long as it produces interesting new results. After 5 decades, there is however the danger that it solidifies in our heads and becomes identified with QFT.

of an action for a particular model is the common ground for starting a joint discourse and people with a good conceptual understanding of QFT know how to use them as an intuitive launching pad for specific conjectures.

Returning to the issue of spacetime embedding and restrictions, what is always mathematically possible is the opposite of an embedding, namely a restriction of higher dimensional QFT to a lower dimensional one; but even in this case the problem of whether the restricted theory has reasonable physical properties remains to be checked on a case to case basis. QFTs formally associated with Lagrangians have the physically correct cardinality of phase space degrees of freedom, but when one restricts such theory to a lower spacetime dimension without a Lagrangian for the restricted theory, the naive argument that this cardinality may be too high for accomodating a physically viable QFT on the lower dimensional side has to be taken serious. This indeed does render holographic projections *physically* questionable, but there is one notable exception in which the degrees of freedom adjusts themselves dynamically to the lower dimension namely the holographic projection from the bulk to its (causal or event) horizon.

This has the consequence that without knowing some feature of the bulk (in the free case the mass of the theory), it is not possible to return with the full information content from a horizon to the original bulk theory since e.g. the knowledge of physical masses gets lost in the projection on a null-surface. In all other cases, in particular in the case of the $\mathrm{AdS}_n-\mathrm{CFT}_{n-1}$ correspondence, one ends up with a mathematically existent, but physically sick situation. A gigantic globalized community whose members produced more than 6000 publications on this topic watches almost 20 years over this conjecture but nobody within the community looked at the degrees of freedom problem. The physical concequences of having too many phase space degree of freedom are well-known: loss of causal propagation, occurrence of Hagedorn temperature or worse).

As long as one thinks about QFT in terms of Lagrangian quantization, there is no problem since classical relativistic Lagrangians (generalizations of Maxwell's theory) have a causal relativistic Cauchy propagation which is inherited by the QFT solution (if it exists), and the thermal physics based on their quantization defines what we consider as a normal physical thermal behavior. But outside the Lagrangian setting the validity of the causal shadow property (the operator algebra in a spacetime region $\mathcal{O}$ is equal to that in its causal shadow $\mathcal{O}''$) as well as normal thermal behavior (thermal states for all temperatures $T > 0$) has to be established. If Jordan would have succeeded to formulate QFT without a Lagrangian (in his terminology without "classical crutches", see next section), he would have run into precisely this problem. The realization of the significant difference between the cardinality in (zero density) QM (finite number of phase space degrees of freedom per phase space cell) and QFT (cardinality per cell is that of a compact or nuclear set [27]) came much later.

The Maldacena conjecture fails precisely on this issue i.e. a physical $\mathrm{AdS}_5$ theory becomes an unphysical $\mathrm{CFT}_4$; this is a structural property without exceptions. It does not help to start the other way around, in this way the higher

dimensional side would suffer from the opposite disease of "physical anemia" i.e. a physically unacceptable lack of sufficient degrees of freedom for finding point- or string-like generating fields of the local akgebras in the $AdS_5$ spacetime. What is not excluded by this physical No-Go theorem for having two corresponding *physical* theories is the possibility that the respective unphysical side may turn out to be mathematically useful to learn something new about the physical side. After all the the mathematical correspondence is a very radical spacetime reprocessing of the same abstract quantum matter. Physics depends on both, the given abstract quantum matter and its order in spacetime.

Returning to the dual model, the strange reinterpretation of a chiral conformal theory as a dual model approximation of an S-matrix permits a vast generalization which confirms the absurdity of its S-matrix interpretation. The natural conceptual-mathematical setting of the duality relation has nothing to do with particle physics of the S-matrix and its on-shell crossing properties. Rather each conformal QFT, independent of its spacetime dimension, is naturally associated with a dual model through its appropriately defined Mellin transform [51]. The sum over infinitely many intermediate poles in the Mellin transform corresponds to the infinitely many terms in a global conformal operator expansion. The residuum has precisely the desired form as required by the dual model or to express it historically correct: the poles and residua are what Veneziano found when (playing with gamma functions) he came across the rules of the appropriately normalized Mellin transforms of a conformal model but thinking that he found an approximate solution of Mandelstam's crossing symmetric S-matrix program. The S-matrix and especially the origin of its crossing properties was terra incognita at that time so that any proposal which was mathematically consistent and fulfilled the at that time accepted idea about crossing could pass as a (nonunitary) approximation of an S-matrix with a given infinite particle multiplet.

Of course the elegant identification with Mellin tranforms of conformal correlations is of a fairly recent vintage and hence the sense of having stumbled upon something at least mathematically very deep by pursuing a phenomenologically motivated program created a lot of excitement of having hit an important structure of particle physics. The only additional requirement which was not part of the original dual model, but rather entered through the string theoretic interpretation, is that the *inner symmetry space of the conformal model should be identified with the spacetime* arena for the action of a unitary representation of the Poincaré group. For anybody to whom string theory has the appearance of being somewhat dodgy, this idea of creating higher dimensional spacetime on the inner symmetry arena of a low dimensional (chiral) QFT is probably the cause of that dodginess.

It is generally not possible to have a representation of a noncompact group as inner symmetries (transformations on field components) since it is known from higher dimensional QFT that the superselection theory requires internal symmetries to be compact. However in d=1+1 the internal symmetry structure is less restrictive and lo and behold, there is really an almost unique solution of this physically unmotivated property, namely (up to a finite number of M-theoretic

modifications) the unique 10-dimensional "superstring", except that *the terminology "string" is misleading*[33] since the infinite component wave function space and its second quantized operators in Fock space have pointlike generators and form what may be called a "pointlike dynamical infinite component field". The addition of "dynamical" is to emphasize that these infinite component free theories are nontrivial in the sense that their algebra contain more operators than those which transform within each level of an infinite mass/spin tower. Without such level connecting operators in the wave function space which determine the mass/spin tower, the infinite component field would be uninteresting. In previous attempts at dynamical infinite component theories the mass/spin spectrum was expected to come from noncompact groups which generalize the Lorentz group [52] but the idea did not work and the project was abandoned.

In order to conclude that the quantization of the bilinearized Nambu-Goto Lagrangian is pointlike generated one would not have to run through any calculation; it would be sufficient to check that in the representation of the Poincaré group resulting from that model there is no Wigner infinite spin component, which for systems with a quadratic Lagrangian is the only way to avoid stringlike covariant theories[34]. The explicit calculation is however useful in order to confirm that the requirement of implementing a unitary positive energy representation representation of the Poincaré group leads to pointlike dynamical infinite component supersymmetric field in 10 dimensions. The interpretation of our 4-dimensional living space as coming from a dimensional reduction of this space remains however in the eyes of the beholders.

But how is such a conceptual mistake about localization possible 40 years after Jordan? Is it that physicists in string theory communities compute correctly but fall behind previous generations if it comes to conceptual problems or is perhaps the problem of localization a very subtle problem on which one could easily slip? It is a bit of both. Despite a significant difference between the Born-Newton-Wigner localization which has to be added to QM for its interpretation, and the intrinsic modular localization of QFT (the modern version of Jordan's causal localization), the issue of localization has remained a rather neglected area of QT. Whereas the embedding of a lower dimensional QM into a higher dimensional one and a Kaluza-Klein dimensional reduction is trivially possible, the holistic aspect of modular localization prevents the realization of this simple-minded idea. What happens concretely in the case at hand is that the quantum mechanical chain of one-dimensional oscillators has no choice to participate in the spacetime localization of a higher dimensional "target". Rather the whole chain is mapped into the internal mass/spin tower i.e. into an internal quantum mechanical Hilbert space over one point which normally accommodates the spin indices; it does not change the holistic localization structure of QFT. To talk about such an internal "string" is certainly not what string theorists mean and also would create terminological problems with situations of true spacetime

[33]The reader should be aware that whenever we continue to use this almost 5 decades old terminology, it is in his historical and not in its physical meaning.

[34]In interacting theories there could be algebraic string generators which lead to states which permit pointlike generation (the Buchholz-Fredenhagen strings [53])..

strings as e.g. electrically charged fields.

The conceptual confusion did not come out of nowhere; for many decades the delusive terminology "relativistic QM" has been used when QFT was really meant. There is no conceptual basis for objects which interact according to Feynman like pictures involving splitting and recombining tubes. One of course take the point of view that these rules can make sense by themselves even if the result of the canonical quantization of the Nambu-Goto Lagrangian are pointlike localized objects. But then one would have to prove (as Stueckelberg did in the case of world lines) that the world tube recipe leads to amplitudes which can be expressed *in terms of states and operators*, the minimal demand for something which is claimed to be of "quantum" nature. Functions and recipes how to combine them are no replacement for a unitarization in terms of operators and states. But this has been tried in vain for many decades as long as string theory exists.

Another metaphoric idea which was already mentioned is the use of special properties[35] of internal symmetries of chiral theories as carriers for fundamental spacetime properties; this is nothing short of mysticism. The rational way for understanding Witten's M-theory conjecture would simply consist in forgetting the the spacetime metaphor and analyze these properties at the place of their origin, namely as special properties of noncompact inner symmetries of certain chiral current theories, but this would be less sexy. But what the heck has the spacetime of Einstein's relativity to do with the greater richness of "inner symmetries" which one meets in nonrational chiral models?

One could of course ignore these somewhat bizarre developments, but there is a point of genuine preoccupation which results from the fact that, as explained in the previous section, the important gauge theories do contain bona fide string-localized interacting fields and may be affected by all these hardened misunderstandings about string-localization localization which do not seem to find an end. This weakening of pointlike field localization in those theories is not directly visible in the infrared divergencies of global "on-shell" observables and their replacement by inclusive cross sections in cases where the latter can be defined as substitutes. Whereas string directions (points in 2+1 dimensional de Sitter space) appear *explicitly* in charge-carrying fields, their occurrence in global on-shell quantities is expected to be more discrete in form of residual dependence on the string directions in the low energy behavior of inclusive cross sections. In the standard gauge description of Yang-Mills theories the infrared divergences are so strongly intermingled with the ultraviolet renormalization that no renormalized correlations of Lagrangian fields have been constructed. One hopes to sort out these problems with a different formalism which takes care of the string-localization of the vectorpotential (previous subsection) and hopefully leads to an understanding of confinement sd the invisibility of certain strings.

[35]Whereas inner symmetries in higher dimensionsn are described in terms of compact groups, chiral theories permit in addition to "rational" theories also irrational ones which are analogs of noncompact symmetries (example; the continuously many charged represntation if chiral current algebras).

The issue of string localization is even more important for zero mass higher spin models e.g. the interactions of s=2 tensor potentials. Furthermore the presently best way to understand renormalizable s=1 massive vectormeson couplings to lower spin quantum matter is to start from e.g. QED with scalars and spinors and force the scalars to undergo a Schwinger-Higgs screening. The latter is similar to the Debeye screening of a Coulomb system in QM, but much more radical in that the Debeye change of range of forces from long to short range corresponds now to the "re-localization" of string-localized to point-localized massive fields.

At no other time there has there been such little foundational progress in particle theory as during the last 3 decades. This can be exemplified in terms of changes how people thought about the gauge theoretic view of massive vectormesons in renormalizable interactions with low spin quantum matter. The Schwinger-Higgs screening mechanism of the 60/70s which has a perturbative formulation starting from scalar QED is nowadays referred to the Higgs mechanism with the "God particle" being the real particle which remains from the screening of the complex charged field to a real neutral field. This is more than a change of terminology since it leads to misinterpretation as a spontaneous symmetry breaking, whereas in reality the symmetry disappeared because the integral over the zero component of the current simply vanishes whereas for spontaneous symmetry breaking it would diverges as a consequence of the presence of a zero mass Goldstone particle. A more profound distinction of what belongs to intrinsic properties of QFT and what is merely part of a computational recipe in a Lagrangian setting would be helpful; unfortunately textbooks do not contain discussions on these issues. The spirit of "superstring theory" could perfectly survive its subject because its main success is the formation of a cordon sanitaire against critical influences around a community.

How much the corrective critical counterbalance has lost its influence on the course of events can be seen by noting the comments of some famous mathematicians when they address a mixed audience of mathematicians and physicists. Among some praise about the depth of the mathematics-physics encounter which finally has been reached thanks to string theory and its derivatives, there is usually also an accolade of the Maldacena result on the gravity-supersymmetric Yang-Mills theory correspondence. To anybody who knows the subject it is evident that they don't know what they are talking about. To get a feeling for the strangeness of this situation just imagine that at the time of the discovery of QM and QFT in 1925 Hilbert and his colleagues from the Goettingen mathematics department would have presented lectures at seminars and conferences in which they would have praised the results of their colleagues. They may have done this later without causing harm after a good part of the new QT had been conceptually and mathematically established. was established. But the Maldacena conjecture is almost 20 years after its vague formulation even further away from a proof than before. It is a good thing to start a study of a problem with an idea of what can be expected. However if this is done by hook or by crook than it becomes an ideology In fact this is a paradigmatic illustration to the kind of physics which grows in the monocultures

of globalized communities without a critical corrective.

## 4 The 1929 Kharkov conference

It is interesting to dedicate a separate section to the 1929 Kharkov conference since this was the high point in the first phase of QFT which in a way was also the culmination of Jordan's career in QFT. It was not only his professional "swansong" in QFT before he, starting in the middle of the 30s, increasingly isolated himself from the international community as the result of his Nazi sympathies, it was also the swansong of Germany's leading role in physics since a few years later German Jewish scientist who made significant cultural and scientific contributions during imperial times and the Weimar republic were forced by the Nazi government to leave their homeland, a brain drain from which Germany never fully recovered.

At this 1929 conference in Kharkov[36] Jordan gave a remarkable plenary talk [54]. In a way it marks the culmination of the first pioneering phase of QFT; but it already raised some of the questions which were only taken up and partially answered almost 20 years later in the second phase of QFT (i.e. in renormalized perturbation theory and its application to gauge theory). In his talk Jordan reviews in a very profound and at the same time simple fashion the revolutionary steps from the days of matrix mechanics to the subsequent formulation of basis-independent abstract operators (the transformation theory whose authorship he shares with Dirac and London) and steers then right into the presentation of the most important and characteristic of all properties which set QFT apart from QM: Commutation Relations in agreement with causal locality and localization of states, as well as the inexorably related vacuum polarization.

Already one year before in his Habilitationsschrift [14] he identified the two aspects of relativistic causality namely the statistical independence for spacelike separations (Einstein causality, commutation of observables) and the complete determination of events in timelike directions (the causal shadow property) as playing a crucial role in the new quantum field theory. This almost modern conceptual viewpoint of QFT was quite a shot away from the "collection of oscillators" formulation in the Dreimännerarbeit where Jordan's main motivation was to show that Einstein's photon contribution in the statistical mechanics luctuations of a blach body radiation gas can also be obtained from a fluctuation calculation on a system of a free photon in the vacuum state. The more general project to generalize this approach to the de Broglie matter waves led to the unified description of matter and light in the form of QFT.

Jordan ends his 1929 conference talk by emphasizing that even with all the progress already achieved and that expected to come in order to clarify some remaining unsatisfactory features of gauge invariance (*Die noch bestehenden*

[36]Landau, after his return from a visit to Copenhagen, went to the university of Kharkov which for a short time became the "Mecca" of particle physics in the USSR. The conference language at that time was still German

*Unvollkommenheiten, betreffs Eichinvarianz, Integrationstechnik usw., duerften bald erledigt sein)*, one still has to confront the following problem: *Man wird wohl in Zukunft den Aufbau in zwei getrennten Schritten ganz vermeiden muessen, und in einem Zuge, ohne klassisch-korrespondenzmaessige Kruecken, eine reine Quantentheorie der Elektrizitaet zu formulieren versuchen. Aber das ist Zukunftsmusik.* (In the future one perhaps will have to avoid the construction in two separated steps and rather approach the problem of formulating a pure quantum theory of electromagnetism (a pure QED) in one swoop, without the crutches of classical correspondences. But this is part of a future tune.)

He returns on this point several times, using slightly different formulations (*....muss aus sich selbst heraus neue Wege finden...)* for a plea towards a future autonomous formulation of QFT which does not have to take recourse to quantization which requires starting with an (at least imagined) classical analog.

These statements are even more remarkable, since they come from the protagonist of field quantization only four years after this discovery. A similar curious flare up of an important idea ahead of its time was the attempt to generalize QED which was rediscovered in the proceedings of a 1939 conference in Warsaw. It contained a talk by Oskar Klein, Jordan's collaborator from the Copenhagen time, grappling with the intricacies of nonabelian gauge theories and their possible applications to particle physics. It seems that the knowledge about QFT in pre-war Europe was more advanced then hitherto thought.

Jordan´s critical attitude towards his own brain child of wave-field quantization has the same philosophical origin as his antagonism to Dirac´s particle approach. Being a positivist, he had no problems with the physical non-existence of a classical structure to be quantized, as long as the quantization was consistent and gave the experimentally verifiable behavior of quantum matter. To him quantization from a uniform setting of classical matter waves was preferable to Dirac's dual program of wave quantization of light and particle quantization for massive matter.

What bothered Jordan about field quantization was the apparent necessity of a parallelism to classical physics which is inherent in the very procedure of quantization, be it that of particles or that of fields. Clearly a fundamental physical theory should stand on its own feet and explain the classical behavior in certain limits, and not the other way around, as in the various quantization approaches. Neither Dirac's nor Jordan's own approach was able to meet this plea for an autonomous approach to QT.

Often the joint work of Jordan and Klein [55] is cited as the inaugural paper on field quantization. This is an easily corrected conceptual misunderstanding. The Jordan-Klein paper is an exposition of the formalism of second quantization in the Fock space setting of multiparticle QM. This has nothing to do with the physical content of QFT, it is just a condensed notation for writing the n-particle Schrödinger equations, which uniquely follow from a two-particle interaction, in terms of one Hamiltonian or one field equation[37] which does the job for all

[37]This Fock space formalism turns out to be quite useful in condensed (nonrelativistic) natter physics at finite density.

particle number n in one sweep. Interacting QFT acts in a Fockspace, but there is no two- or higher- particle relativistic equation from which it arises through "second quantization", in fact apart from free fields which arise from Wigner one-particle representations in a functorial way, there is no setting from which QFT arises by second quantization; the latter is simply incompatible with causal localization-preserving interactions.

The only structure which resembles the Jordan-Klein Schrödinger field formalism occurs in the absence of interactions when the one-particle spaces may be classified in terms of Wigner's representations of the Poincaré group. In that case one may define a functor which leads from the modular localized subspaces of the Wigner representation space to the net of local von Neumann algebras as the "second quantization functor" as explained in the previous section. But such a terminology would be limited to interaction-free theories.

In retrospect it is clear that an autonomous approach to QFT had no chance before the formalism of interacting field quantization was fully developed to the level of the conceptually sophisticated renormalized perturbation theory, as in the work of Tomonaga, Feynman, Schwinger and Dyson in the end of the 40's. Only after this seminal work, the mathematical status of the inherent singular nature of pointlike fields became gradually understood, and arguments which permit to avoid infinities in intermediate computational steps were proposed. An autonomous formulation of QFT which is capable to classify and to construct models has been in the making since the beginning of the 60s; it had a slow start but gained increasingly steam during the last two decades. What is referred to as LQP [27] or AQFT is a set of physically motivated and mathematically well-formulated requirements which allow to derive interesting general consequences. In d=1+1 one knows additional structures which permit a partial classification and construction of models. The progress obtained during recent years nourishes the hope that a complete knowledge of autonomous QFT including a classification of models and a proof of their mathematical existence is possible in the realistic case of d=1+3. None of these recent attempts in LQP can be traced back to Jordan's 1929 plea for a QFT; the time lapse is too large and world war II has interrupted a continuous development. Nevertheless it is encouraging to know that the first pleas for an autonomous understanding of QFT are even older than renormalization theory, deep ideas have their harbingers before they take roots. Some more remarks about the post Jordan development of these concepts will appear later.

Jordan's expectation about a rapid understanding of the "imperfections of gauge theories" at the time of his Kharkov talk may have been a bit optimistic since the Gupta-Bleuler formalism (much later followed by the more general BRST approach) only appeared 20 years later. But it is interesting to note that for Jordan gauge theory was an important issue already in 1929. His view of gauge theories was, as that of the majority of contemporary "applied" quantum field theorists, just taken over from the classical use of vectorpotentials where they are an indispensable computational tool (example: Lienhard-Wiechert potentials). It is questionable that he had a foundational QFT view as that explained in section 3.2 which sees quantum gauge theory as the consequence

of resolving a fundamental clash between the quantum Hilbert space structure with the existence of pointlike vectorpotentials which either requires to abandon the Hilbert space structure in order to save an initially unphysical but technically well-known perturbative formalism, or to confront dealing with stringlike potentials as in subsection 3.2.

Finally one should add one more remark which shows that Jordan and Dirac had a very similar taste for what both considered relevant problems. In 1935 Dirac presented a beautiful geometric argument which establishes that it is possible to introduce a magnetic monopole into quantum electrodynamics as long as the strength of the monopole times the value of the electric charge fulfill a quantization law. In the same year Jordan came up with an interesting very different algebraic argument for the magnetic monopole quantization which he based on the algebraic structure of bilinear gauge invariants [57]. He published a short note in Zeitschrift fuer Physik at a time when its international reputation had suffered from the deteriorating political situation (see first footnote). Hence it is not very surprising that this paper was hardly noticed. The independent re-discovery of its main content (including Jordan's tetrahedron argument) by R. Jackiw [58] certainly shows that the problem remained non-trivial and interesting almost 70 years later. The problem of how these kind of arguments have to be amended in order to take account of renormalization has according to my best knowledge not been satisfactorily answered. Jordan's monopole paper is one in series of publications in which gauge invariance based on exponential functions of line-integrals over vectorpotentials play a central role (see next section). It is clear that what Jordan really wanted to achieve was a gauge invariant formulation of the basic dynamical equations of quantum electrodynamics. This he did not achieve, but attempts 30 years later, in particular by Stanley Mandelstam, also failed. Although gauge theory aims at gauge invariant observables and as such fulfills Heisenberg's dictum that the use of non-observables must be avoided since, it does use unobservable negative metric ghosts in intermediate steps. Their use is reminiscent of a "catalyzer" in a chemical reaction which can be removed after having done its job. Whereas their mode of action in chemistry is well-understood, there is no fundamental understanding why one needs such ghost catalyzers for the description of interactions involving higher spin equations. Their use is obviously helpful for the construction of gauge invariant local observables, but the necessarily nonlocal physical charge-carrying fields remain outside the standard gauge formalism.

Since Jordan left the area of QFT in the middle of the 30s and became disconnected from the important post-war discoveries, we do not know how he would have reacted to the amazing progress brought about by renormalization theory. Probably, as most of the leading particle theorist of the thirties, Jordan would have expected that only a revolution in the conceptual foundations could save QFT as a theory of relativistic particles. In that case he would have been deeply surprised that the post war progress resulted from a careful conceptual distinction between formal and physical (observed) Lagrangian parameters, as well as by a systematic replacement of the old quantum mechanic-inspired formalism by a more intrinsic field theoretic relativistically covariant setting and

a lot of hard computations. In the words of Weinberg the success was achieved in a rather conservative manner [25].

In fact renormalization theory may have been too conservative in order to fit Jordan's radical expectations; in strange contrast to his reactionary political stance, in physics Jordan was a visionary revolutionary.

## 4.1 Local Quantum Physics and Jordan's ideas about a future QFT

30 years after Jordan's state of the situation presentation in Kharkov a conceptional renewal of QFT was set in place which nowdays is known as local quantum physics (LQP) or algebraic QFT (AQFT). Its immediate predecessors were Wightman's formulation of QFT, which gave rise to the first mathematical treatments [38], and the LSZ setting linking asymptotes of fields with particles [27]. But the project which fits best Jordan's quest for an intrinsic formulation without any quantization ties to a classical parallelism is that which Haag outlined for the first time at a conference in Lille in 1957 [59]. By that time Jordan had become the unsung hero of QFT and there was certainly no historical connection of the new attempts to get a more intrinsic access to local quantum physics (LQP = QFT with the emphasis on the causal locality principle) with the old prophetic pronouncements at the Kharkov conference.

Up to the 60s a lot of things had happened in QFT. There was a much better understanding of the true nature of quantum fields. In contradistinction to classical fields and second quantized Schrödinger fields (as von Neumann showed it is not necessary to know the delta function in QM) the new distribution theory of Laurent Schwartz of the 50s was essential in order to understand the conceptual position of quantum fields within QT. The so called ultraviolet divergencies of QFT which led almost to its abandonment were partly cause by treating fields as operators and not as operator-valued distributions. In the 60s it became clear in the work by Epstein and Glaser [62] that a perturbative classification and construction of renormalizable models can be given in the setting of time-orderd correlation functions of fields in terms of the short distance scaling degrees of local polynomials in the free fields describing interactions. If the short distance power counting leads to a degree less than 4 (in d=1+3), than the perturbative series can be written in terms of a finite number of interaction parameters independent of the order and the theory is called renormalizable. Different from standard quantum mechanical inspired approaches, which try to renormalize infinities which were the result of incorrectly handled singular aspects of pointlike fields (operator-valued distributions) and their composites, the E-G method iterates the zero order input according to the locality principle and stays in a multi-particle Fock space. This was the kind of perturbative formulation which fits best Haag's impressive design and Jordan's dream.

An important aspect of that construction was the explicit knowledge of the equivalence class of all fields which are Einstein causal relative to the free field. This local equivalence class consisted of the pointlike Wick polynomials of a free field. This and other structural results coming from causal localization

(modular localization in the more recent mathematical setting) permitted to base perturbation theory on physical principles rather than on approximating concrete operators as in QM. But the results were of course the same as that of the cutoff or regularization methods, the only reason for mentioning the finite methods is that some people (especially string theorists) believe that QFT is beset my unavoidable ultraviolet problems whereas the real problem of perturbation theory is the number of coupling parameter which limit the predictive power and which render nonrenormalizable interactions useless. Later it became clear that renormalized perturbative series never converges. This implies that perturbation theory has no explanatory power concerning the existence of these models; the best one can hope for is that there is a notion of asymptotic convergence in the limit of vanishing couplings which could be behind the often spectacular experimental agreement in QED and weak interactions.

The structure of local equivalence class of (infinitely many) pointlike fields suggests that fields in such a class have, different from classical fields, no individual distinction from each other apart from their charge; locally equivalent fields which carry the same superselected charge create the same particles and lead to the same scattering theory (interpolating fields for the same particle scattering). Hence the use of different fields in such a class in a fixed QFT is like the deployment of different systems of coordinates in geometry. Haag build his LQP on the idea that it is not the individual field but a spacetime indexed net of operator algebras. In this way the individual fields disappeared and only the concept of charge remained with the algebras; similar to coordinates in geometry they now played the role of generators of algebras of which there were infinitely many. With the spacetime-indexed net of algebras as defining a QFT, the confusing plurality of QFT disappeared and instead the question arose whether this setting describes all physical phenomena which were previously described in terms of individual fields. The answer to this question is positive if one attributes a preferential status to conserved currents which according to Noether's theorem already had a distinguished status in the classical theory. One can show that also in the algebraic setting conserved currents can be constructed from a (localized version) of symmetries of the QFT and this construction can be generalized to spontaneously broken symmetries.

It is much more difficult to convince quantum field theorists that the intrinsic local operator algebra approach is the most suitable in order to take on the remaining difficult problems of particle theory whereas they except immediately that in mathematics the coordinate-independent intrinsic formulation of geometry is the most adequate one. Often those particle physicists who have internalized the intrinsic formulation on the mathematical side (fibre bundles, cohomology,...), turn out to be the fiercest defenders of the status quo if it comes to the conceptual-mathematical setting of QFT.

For the first three decades the intrinsic setting of algebraic QFT has been predominantly applied to the solution of deep structural problems as e.g. the quantum origin of the notion of internal symmetries which for the first time entered nuclear physics through Heisenberg's isospin. This was not part of classical physics (although often physicists read such properties back into the

classical setting) so it was natural to ask the question about its origin. In Haag's setting of local quantum physics the question took on the more specific form: what is the role of locality in de-mystifying internal symmetries? The answer was that the localizable representations of an observable net (the localizable superselection sectors) have the structure of a dual of a compact group and the best way to present this situation consists in defining a field algebra consisting of Bose and Fermi fields on which this internal symmetry group acts such that the original charge neutral observable algebra re-emerges as the fixpoint algebra [27][63]. Being a structural theorem, this does not distinguish a particular compact group (in fact it can be shown that all compact groups can occur). Somewhat exaggerated but logically correct one may say that if groups were not discovered in other purely mathematical contexts way back, the quantum locality principle would have also spotted this structure.

QFT is different from any other theory in physics, including QM, in that one has not been able to come up with nontrivial example involving interactions. This is a problem Jordan must have been aware of because what he too optimistically called "neutrino theory of light" can be considered as a result of searching in d=1+1 for well-defined nontrivial models. As mentioned renormalized perturbation theory did not solve the problem of finding interacting models either since there are rigorous statements which show that the there are no circumstances under which the perturbative series can converge. LQP only postponed but did not suppress the longing for an existence demonstration of a nontrivial illustration for interacting QFT. Finally, starting in the 90s, some ideas which were endogenous to LQP appeared [64]. They led to successful existence proofs for QFT from the class of factorizing models [50].

These existence proofs follow a different logic from that suggested by the old functional analytic Glimm-Jaffe arguments which were limited to theories with the same short distance behavior as free ones (superrenormalizable), such couplings only exist in d=1+1; it is also totally different from what a functional integral representation would suggest. The idea is to start as far as possible from pointlike fields i.e. with objects which have the least amount of vacuum polarization i.e. with the coarsest localized objects because they have the least amount of vacuum polarization. These are generating operators of wedge localized algebras. The mentioned class of models is distinguished by rather simple properties of wedge generators which lend themselves to a classification. The compact localized algebras which have the full vacuum polarization and whose generators are the pointlike localized quantum fields are then obtained by intersecting wedge algebras and the hard part of the existence proof consists in showing that these intersections are nontrivial and act on the vacuum in the standard manner.

Even the perturbative approach, which has the closest contact with measurements and phenomenological ideas, is incomplete. More precisely it is a closed subject only in case of pointlike interactions but not in case of gauge theories and interactions involving massless higher spin representations. In that case one needs a generalization of the Epstein Glaser approach to stringlike free fields since this is the only possibility to understand the origin of string localization

charged matter fields whose's delocalization comes about through interactions with string-localized free vector potentials.

It is interesting to note that the factorizing models are classified by elastic S-matrices. Since the cardinality of factorizing S-matrices is much bigger than that of interacting Lagrangians, most existing factorizing QFTs have no Lagrangian name i.e. the Lagrangian quantization setting does not exhaust the possibilities of models of QFT. This illustrates the validity of Jordan's point that the use of classical crutches in the form of Lagrangian quantization may turn out to be too narrow.

# 5 "Bosonization" instead of "neutrino theory of light"

Starting around 1935 Jordan began to publish a series of papers under the title "On the neutrino theory of light" [65]. The idea that photons may be bound states of $\nu$-$\bar{\nu}$ was not entirely new since de Broglie had vaguely formulated a similar thought but without presenting an argument. The statistics of particles in terms of commutation relations of fields was still an unaccustomed subject and therefore problems which nowadays are considered as part of kinematics to be done away with in a few lines, at that time filled the main part of a paper. There was the general belief from QM that it is sufficient to illustrate an idea in a low-dimensional QFT; we know thanks to Wigner's work on Poincaré group representation theory of particles that this is incorrect. This explains to some extend why Jordan's contemporaries had no problem with the fact that he took a two-dimensional model instead of arguing in the realistic setting of 4-dimensional spacetime. Jordan started from a two-dimensional massless Weyl fermion, his neutrino model, from which he formed a bosonic current and its potential which was then his two-dimensional analog of the photon field.

His model amounts to what since the 70s is called *bosonization* and *fermionization* refers to its inverse; both procedures only work in two dimensions and shed no light on a higher dimensional physical neutrino theory of light. But although Jordan was wrong in his central claim, he discovered at least the mathematics of an interesting new structure for which however there was no demand in QFT prior to the 60s. Before the presentation of Jordan's model in more detail, some additional remarks on the state of QFT at the time of Jordan's participation are in order.

Using a modern notation and terminology his main points in [67] become more accessible. Starting from a $u = t + x$ chiral free fermion $\psi(u)$ one may define the u-component of a chiral current

$$\begin{aligned} \psi(u) &= \frac{1}{(2\pi)^{1/2}} \int_0^\infty dp(e^{ipu}a(p) + e^{-ipu}b^*(p)) \\ j(u) &=: \psi^*(u)\psi(u) : \ = \ \partial_u V(u), \quad u = t + x \end{aligned} \tag{22}$$

The u-lightray component of the spinor has a $v = t - x$ counterpart which is not

needed here, but would be required for the massive spinor which depends on both $u$ and $v$ (for which however the commutation properties are less simple). The double dot denotes as usual the ordering in which all annihilation parts appear in the right of the creation operators (with a sign factor for each fermion commutation). The only commutation relation one has to know in order to compute all the others is that between (Wigner) momentum space creation and annihilation operators are the standard ones $\{a(p)a^*(p')\} = \delta(p-p')$ and similarly for the $b$.

The surprise is the result of the $j(u)$ commutator. On would naively think that the commutator of an expression which itself is bilinear in fermions would contain in addition to a c-number term (complete contraction) also a bilinear term with one contraction; but against naive expectation this operator term (which would be present in higher dimensions) is absent for 2-dimensional massless fermions [68]. In fact this is the only case where this simplification occurs, in all other cases the bilinear current of a Fermi field does not fulfill canonical commutation relations. This tricky aspect which was which was correctly handled by Jordan was not understood by his contemporaries, in particular it brought him into a conflict with Fock [69]. In the abstract of the above cited paper Jordan vigorously (and correctly) refutes Fock's critique. Nobody at that time seemed to have found it problematic to draw physical conclusions from d=1+1 zero mass "neutrinos" and "photons" about the real d=1+3 situation; what was not accepted was that the the current from a free massless spinor in d=1+1 is a canonical field, just opposite from what we know nowadays!

It is worthwhile to write the form of the current commutation relation

$$[j(u), j(u')] = c\delta'(u-u') \tag{23}$$

Jordan performed his calculation in the filled Dirac sea i.e. in the charge symmetric prescription; the use of the *hole theory* would have caused a serious confusion in particular in such calculations. This derivative term in a commutation relation was equivalent to the presence of the so-called Schwinger terms of 1959 [70] i.e. derivative of delta functions in the mixed space-time components of currents in any dimension.

The cited paper of Jordan also treats the inversion of bosonization, namely the re-fermionization starting from the potentials $V(u)$

$$\begin{aligned} \Psi(u,\alpha) &\equiv e^{i\alpha V(u)} = e^{i\alpha\int_{-\infty}^{u} j(u)du} \\ [j(u), \Psi(u',\alpha)] &= \alpha\delta(u-u')\Psi(u',\alpha) \end{aligned} \tag{24}$$

The case $\alpha = 1$ which leads back to the zero mass canonical fermion with scale dimension 1/2. From a modern point of view these fields are charge-carrying fields of charge $\alpha$ associated with the current operator $j(u)$. Each charge defines a superselection sector i.e. there are continuously many. The $\Psi$ are fields which turn out to be conformal covariant and therefore possess a scale dimension which is proportional to $\alpha^2$ (the proportionality constant depends on the normalization of V). Together with the anomalous scale dimension the charge $\alpha$ determines also

the anomalous (conformal) spin and through it the statistics which turns out to be "anyonic" i.e. that associated with an abelian braid group representation. There is one value of $\alpha$ for which the conformal spin is $1/2$ and the anyonic commutation relation becomes fermionic; this is the value which Jordan used for re-fermionization and for which he showed that the $\Psi$ coalesces with the original fermion $\psi$. Hence starting from a chiral Fermion $\psi$ one passes to a chiral current $j$ which turns out to be the derivative of an (infrared divergent) free field $j(u) = \partial_u V(u)$ which in turn through (24) leads back to a free Fermi field $\Psi(u, \alpha = 1)$ in a different veil. This passage is nowadays referred to as bosonization/fermionization. It is limited to d=1+1 and in the above form also to the chiral (the u,v lightray) components of a Fermion or a current.

The model is identical to that which Jordan used in the Dreimaennerarbeit to support Einstein's fluctuation formula namely the solution of the wave equation

$$\begin{aligned} &\partial_\mu \partial^\mu \Phi(t,x) = 0,\ \Phi(t,x) = V(u) + V(v),\ j(u) = \partial_u V(u),\ j(v) = \partial_v V(v) \quad (25) \\ &T(u) =: j^2(u): \quad T(v) =: j^2(v): \qquad T(x,t) = T(u) + T(v) \end{aligned}$$

Here the u and v dependent objects are independent singular operators and the $T$ denote the lightray components of the energy momentum tensor. Instead of considering these objects on their full u,v lightrays where they possess their complete 3-parametric Moebius spacetime symmetry on each lightray, Jordan quantized the system in a an interval of length L and studied the T-fluctuation in a smaller interval I. The L interval quantization removes the infrared divergence of the $V(u)$, although this would not have been necessary since only the infrared well-behaved $j$ and $T$ enter Jordan's energy fluctuation calculation. But quantum physicists (not only at Jordan's time) preferred to deal with sums rather than integrals even if it meant loosing helpful symmetries and shifting problems to other places. As a side remark, the word string in Jordan's contribution is synonymous with a one-dimensional periodic quantization box and has nothing to do with string-localization or string theory since the fluctuation computation can be formulated in terms of the pointlike localized current operator $j(u)$. The fluctuation region is an interval of size $I$ inside the string interval of length $L$.

The modern derivation of Einstein's fluctuation formula would consist in computing the localization energy associated to the interval I on the full u or v lightray. In doing this one can bypass quantum mechanical arguments and use concepts from modular localization which are intrinsic to QFT and have no counterpart in QM. Since the restriction of the vacuum state to the operator algebra of an interval $I$ is known to be a singular[38] thermal KMS state with the generator of the $I$ preserving dilation being the "modular Hamiltonian", one has to approximate this state (as in the case of the thermodynamic limit) by a sequence of Gibbs states. Modular theory provides a canonical way for doing this: the split property [16]. But although the modular localization theory assures the existence of these Gibbs density states which converge against a

[38] The thermodynamic limite state of an increasing sequence of Gibbs states for V$\rightarrow \infty$ is the only KMS state in QM. It leads to finite thermal correlation functions but admits no description as a denslity state (the partition function diverges).

singular KMS state, there is yet no easy calculational approach for the sequence of Gibbs like density matrix states. This step of approximating the singular KMS state for the restriction of the vacuum to the sharp localized interval by a sequence of density matrix states which are sharp- in I but fuzzy-localized inside the $\varepsilon$-collor at the end points corresponds precisely to Heisenberg's "fuzzy boundary" [5] where $\varepsilon \to 0$ corresponds to the return to the sharp boundary (which in chiral models causes a logarithmic divergence but in higher dimensions contains additional inverse powers [66]). The localization energy and its square fluctuation has not been calculated along these line, only the somewhat simpler dimensionless localization entropy has been derived. In the one-dimensional case of QFT on the lightray there is an exact relation between the standard heat bath state associated to the tranlative Hamiltonian and the thermal aspects caused by modular localization in an interval $I$ (the inverse Unruh effect [16][66]). One expects that the leading behavior of the energy fluctuation can be derived similarly. Since Einstein uses rather general properties of thermal averages and the density matrix state from the restriction of the vacuum to the operators localized in the interval $I$ with fuzzy boundaries has the form of thermal state Gibbs state, where the role of the heat bath temperature kT in Einstein's case is played by the "geometric" modular temperature [66].

The main reason for mentioning these ideas based on modular localization in connection with Jordan's quantum mechanically inspired calculation and its recent improvement [5] is to emphasize that, although QFT was discovered in 1925, the cutting of its umbilical cord to QM and the discovery of its conceptual and calculational autonomy was a long-lasting process. Without understanding QFT from its intrinsic conceptual modular localization structure, which includes in particular the thermal aspects of QFT localization, the relation between Jordan's calculation in the vacuum state and Einstein's thermal energy fluctuation in a heat bath state would have remained a plausible analog but surrounded by an air of mystery which no quantum mechanical improvement can fully remove. A restriction of the vacuum state in second quantized Schrödinger quantum mechanics to an interval would just remain a vacuum on that localized tensor algebra and not manifest itself as a singular KMS state; it is not the second quantized field formalism but rather the aspect of modular localization which distinguishes between QM and QFT see [16]. These new localization-based concepts do not only remove the loose conceptual ends of the past, but they also have led recently (for the first time in the history of QFT!) to the rigorous construction of certain two-dimensional models [64][50] with methods which are far removed from the shared common stock of (Lagrangian, functional integral) quantization methods of QM and QFT. It seems that the heavy investment into Haag's LQP [27] project dedicated to the discovery of the foundational properties of QFT, which started in the early 60s and gained pace in the last 3 decades, is now bearing its first fruits and begins to revolutionize QFT in the direction of its autonomous understanding.

Returning to the work on bosonization/fermionization; what one does not find in Jordan's papers [67] are calculations of states or correlation functions. With QFT at that time still being tied down to the formalism of QM it is

doubtful that anybody at that time would have had the conceptual resources one needs to calculate the correlation functions of the $\alpha$-charged fields

$$\langle\Psi(u_1,\alpha_1)\Psi(u_2,\alpha_2)...\Psi(u_n,\alpha_n)\rangle \neq 0 \; only\ for \; \sum_{i=1}^{n}\alpha_i = 0 \tag{26}$$

and extract the charge superselection rules from the infrared behavior of the $V(u,\alpha)$. This certainly cannot be understood in terms of quantum mechanical computational rules. A more demanding method based on the DHR (Doplicher-Haag-Roberts) superselection method of algebraic QFT which is a consequence of modular localization can be found in [72].

As already mentioned, at the beginning of QT there was a widespread belief that one only needs to present an illustrative mathematically controlled model in two-dimensional spacetime dimension, its validity in d=1+3 would then follow by analogy. Whereas this is true for most problems of QM, this is not so in QFT. The properties of relativistic particles are dependent on the spacetime dimensions; there is no good analog of photons and neutrinos in d=1+1. As mentioned nobody at the time criticized Jordan's two-dimensional "neutrino theory of photons" for its lack of validity in the realistic case. Only with the arrival of Wigner's intrinsic representation theoretical approach to particles this dimensional dependence begun to attract notice. But Jordan's contemporaries were deeply suspicious of the d=1+1 bosonization of a Fermion current; as mentioned Fock wrote a counter paper [69] in which he claimed that Jordan made a computational mistake.

There was the feeling that with the neutrino theory of light Jordan had let his imagination go overboard. Hence a little rubdown was in store. It came as a carnavalesque "Spottlied" with the following text (the melody is that of Mack the Knife) [74]:

| | |
|---|---|
| "Und Herr Jordan | "Mr. Jordan |
| Nimmt Neutrinos | takes neutrinos |
| Und daraus baut | and from those he |
| Er das Licht | builds the light. |
| Und sie fahren | And in pairs they |
| Stets in Paaren | always travel. |
| Ein Neutrino sieht man nicht." | One neutrino's out of sight". |

One would suppose that this rather good humored song was presented at the end of a conference or during a conference dinner, but Pais does not comment on this. Insofar as the mock song refers to the misleading title it is unintentionally correct, since they thought that the error was in the presentation of the bosonization/fermionization idea in which the paper was not only correct but even far ahead of its time! The mistake Jordan and his taunters made was to take for granted that each phenomenon in low dimension permits an extension to higher dimension; this is true in QM but not in QFT.

Our modern viewpoint on this issue is that although the photon cannot be viewed as a $\nu$-$\bar{\nu}$ bound state, an interacting QFT is forced (essentially by its

locality principle) to follow the local quantum physical adaptation of "Murphy's law" [16]: what is not forbidden (by superselection rules) to couple does couple!

Applied to the problem at hand this means among other things

$$\langle 0 \,| F_{\mu\nu}(0) |\, p, \bar{p}' \rangle^{in} \neq 0 \tag{27}$$

In words: the formfactor of the photon field between a $\nu$-$\bar{\nu}$ state with momenta $p$ and $p\prime$and the vacuum is nonzero (but as a result of the presence of week and electromagnetic interaction it is extremely tiny). The label in/out on particle states of formfactors is important because the connection between particles and fields in the presence of interactions was not at all understood at the time of Jordan but starting from the 60s we know that interacting QFT has no particle at finite times, they only have an asymptotic reality for infinite times in the sense of scattering theory. This is sufficient to secure their existence as states in the physical Hilbert space, but not for generating particles in compact spacetime regions. One consequence of what has been metaphorically referred to as "Murphy's law" of QFT is *nuclear democracy* namely that the quantum mechanical hierarchy between elementary particles and bound states disappears in QFT; the only remaining hierarchy is that between fundamental and fused "charges". The localization structure of the vacuum representation of the observable algebra contain the informations which are needed in order to construct all non-vacuum superselection sectors and combine them into a field algebra on which a symmetry algebra acts in such a way that the observables re-emerge as the fixpoint algebra under internal symmetry transformation [73].

At this point it may be helpful to take another short break and remind the reader of the change of content of meaning of the term "QFT" during the passage of time. Within the first two decades it was often used in the sense *QM of systems of infinite degrees of freedom*, in particular if those infinite degrees of freedom result from Fourier decomposition of operator-valued space(time) dependent fields. However the mere change of formalism from a two particle Schrödinger equation to a multiparticle formalism as done in the work of Jordan and Klein [55] would nowadays not pass as a genuine illustration of the spirit of QFT, at best it serves to illustrate some aspects of its formalism. Already at the time of Jordan the "second quantization formalism" received mocking comments from some quarters as: "Why quantize something a second time which was already quantum". A more useful well-known comment by Edward Nelson settled this issue once and for all by stating: "second quantization is a functor and quantization is an art".

An intrinsic characterization of QFT is based on (physically) causal or (mathematically) on modular localization. Here "causal" refers to the kind of relativistic causality of finite velocity Cauchy propagation for hyperbolic equations. This can be formulated in an intrinsic way which does not refer to quantization. The characteristic phenomenon of vacuum polarization as discovered by Heisenberg in studying the relation of conserved currents and their charges in free field theories [18] and in the presence of interactions in the work of Furry

and Oppenheimer[39] [19] is the characteristic phenomenon of causal localization which has no counterpart in the localization arising from a position operator in QM. An analog to vacuum polarization (particle-hole pairs) is also encountered in case a second quantized formalism is used in a ground state of finite density (the Fermi surface); this explains why the formalism of QFT in certain cases has a useful analog in solid state physics.

What sets QM and QFT apart in the most dramatic way are their totally different localization concepts [16]. As a result the local operator algebra in QM defined in terms of the second quantized Schrödinger fields are of the same type as their global counterpart namely type $\mathrm{I}_\infty$ von Neumann factor algebras; the local algebras $\mathcal{A}(\mathcal{O})$ of QFT in causally complete regions are radically different namely hyperfinite type $\mathrm{III}_1$ factors, shortly called "monads" since they are all isomorphic to such a monad in [16]). As a consequence there is no tensor factorization between a localized operator algebra and its commutant which is localized in the spacelike complement, even though these two algebras commute! From this follows that there is no notion of entanglement in this situation; i.e. the point of departure of quantum information theory between spatially independent systems has been lost in the setting of QFT. The restriction of the vacuum state to a local algebra $\mathcal{A}(\mathcal{O})$ is not a vacuum of the smaller region as it would be in QM and as one might naively expect in QFT. Rather such a reduced vacuum is a so-called KMS state i. e. a thermal state with respect to a modular Hamiltonian which is intrinsically associated to the localized algebra together with the vacuum. This phenomenon was first perceived in the context of localization behind event horizons in the case of black holes and the Unruh Gedankenexperiments. In that case the modular Hamiltonian has a physical significance in terms of Killing symmetries and the thermal aspect accounts for the Hawking radiation and the Unruh Gedankenexperiment in which the local algebra is the wedge-localized algebra $\mathcal{A}(W)$ with is causal upper horizon $H(W)$.

Thanks to the modular localization theory we now know that all these observations are consequences of the monad structure[40] of the local algebras [16], the only type of algebra which is consistent with causal localization and which leads to thermal manifestation from modular localization. A characteristic feature of this algebraic structure is the fact that sharp localization causes infinite vacuum polarization at the localization boundaries which in turn leads to localization-energy or localization-entropy with respect to the vacuum to be infinite instead of zero as in QM. The quantum mechanical analog is a global KMS state obtained from thermal Gibbs states quantized in a box of volume V in the thermodynamic limit $V\to\infty$; such states correspond to an heat bath in an open system and are not related to localization. The algebra changes its nature in this limit and the box quantized type I standard quantum mechanical algebras

[39] In the presence of interactions all fields, elementary and composites, applied to the vacuum create in addition to the desired particle infinitely many particle/antiparticle pairs (polarization cloud).

[40] The perhaps conceptual distance between QFT and QM arises in the much stronger relational (holistic) characterization of QFT in terms of the modular positioning of a finite number of monads in a joint Hilbert space [16].

passes into a limiting "monad" with a volume diverging energy/entropy [27]. There exists an analog to the thermodynamic limit in the local QFT case: the "split limit", whereby one approximates the local monad by a sequence of type I factors which are localized inside the an augmented double cone $\mathcal{O}_\varepsilon$ with $\varepsilon$ being the size of a collar around $\mathcal{O}$ within which the vacuum polarization can attenuate; the part in the collar is only "fuzzy" (not sharp) localized in turns out that the monad algebra $\mathcal{A}(\mathcal{O})$ together with the collor results in a quantum mechanical type $\mathrm{I}_\infty$ factor. The close relation between the thermodynamic volume divergence and that of the split limit $\varepsilon \to 0$ actually amount to much more than an analogy [66].

The contribution by Jordan to the Dreimaennerarbeit which is rightfully heralded as the beginning of QFT would even nowadays serve as the perfect example to illustrate all the points above which separate QFT from QM and which have served as an never ending source of new conceptual ideas. A modern discussion would start from the system of algebras generated by a current as in (25). A restriction of the vacuum state to a finite interval leads to a singular KMS state with infinite entropy/energy.

In fact it was Heisenberg who pointed out this vacuum polarization caused divergence problem to Jordan in 1931 (see [5]) before he wrote his famous paper about boundary-caused vacuum polarization in partial charges [18]. Nowadays we know that these warnings against retaining only a few frequencies (or occupying levels in the spirit of QM) were well-founded because QFT has a *holostic* structure. Cases where such simple arguments go completely wrong are known in connection with simple minded level occupation methods in order to compute the cosmological constant (see also [10]). Here *holistic* implies that one cannot understand the infinite degrees of freedom of a quantum field theoretic system by separating it into finite sets. Jordan's problem of the energy fluctuation in a finite interval of a chiral theory localized on the lightray is particularly suited to be treated with the intrinsic and holistic modular localization methods of QFT; in chiral models the leading coefficient of the localization entropy[41] in $\varepsilon \to 0$ can be calculated exactly. The thermal nature of the interval-restricted vacuum would have been of great conceptual and calculational help, but not having these (intrinsic to QFT) concepts available at this early time, Jordan used quantum mechanical methods which consisted in performing a frequency decomposition of the field $u(t, x)$ and treating the resulting system of infinitely many oscillators as a problem of QM with infinite degrees of freedom[42]. A treatment in the same quantum mechanical spirit but with Jordan's tacit assumptions being spelled out more explicitly is contained in [5].

Problems as divergencies caused by sharp localization, which only were understood recently, had to be dealt with in a somewhat artistic manner in Jordan's time. In fact the reason for a certain critical distance which his coauthors

[41]The dimensionless entropy is the simplest quantity which shows the thermal behavior of localization onto a subinterval.

[42]One note of caution, the use of "string" in the old papers has nothing to do with string-localized fields since the underlying field in Jordan's model is pointlike. The "string" refers to the linerar version of a box-quantization.

Born and Heisenberg kept to his more revolutionary colleague was related to some ad hoc assumptions and artistic jumps in Jordan's calculation which resulted from the fact that short of a conceptual framework for how to deal with problems of QFT one had to rely on less than perfect quantum mechanical methods. It would be a misinterpretation of the situation to view Born and Heisenberg as being too conservative with respect to their brainstorming young colleague; in contrast to Heisenberg's discovery of QM less than a year before, Jordan's QFT did not come with ready to use conceptually supported computational tools. In fact it took more than 80 years of conceptual research to find the present setting.

Even at the risk of making statements which sound exaggerated, it is just the fact that the quantum mechanical approach in Jordan's work and in its recent improved review [5] did not solve the *thermal side of the conundrum* which is the strongest indicator that *Jordan did not just discover a another model of QM with infinite degrees of freedom but rather came across a completely new conceptual setting.* The insufficiency of the quantum mechanical operator techniques and the birth of QFT are two different sides of the same coin and the critique of his colleagues, in particular of Heisenberg, who a few years later discovered the quantum field theoretical vacuum polarization, was well aimed.

The existence a strong critical counterweight against a new speculative idea is the best indicator of a healthy particle theory. It is the disappearance of this counterweight and the demise of the "Streitkultur" since the 80s when those, who by their reputation and articulation were the natural candidates to fill this role became instead the propagandists and salesmen of new speculative ideas that led to a conceptual deterioration and a schism within particle theory.

Unlike QM where its discovery almost immediately led to various computational settings within mathematical control as well as a rapid understanding of its underlying philosophy, this process took a much longer time in QFT owing to its extraordinary subtlety and richness. The great perturbative progress in the post world war II renormalization theory ended with impressive numerical successes, but there was also the sobering conclusion that the diverging perturbative series has at best the status of asymptotical convergence for infinitesimally small coupling. This is certainly not sufficient in order to secure the existence of a model. Only very recently the setting of modular localization, an extremely different construction far removed from Lagrangian quantization and functional integrals [32][64][50], led to the existence of a certain class of two-dimensional models with a nontrivial factorizing S-matrix. It is interesting to note that these methods use the holistic modular localization ideas in an essential way i.e. the same concepts which are important in the complete solution of the Jordan-Einstein fluctuation conundrum [16][66].

Relativistic causality entered Jordan's work for the first time explicitly in the Jordan-Pauli calculation [56] of the photon two-point function and played an important role in his 1929 review of the first phase of QFT [54]. But it would be far-fetched to conclude from these occasional flare ups of new ideas that causal locality was clearly seen as the characteristic feature of QFT that sets apart QFT from QM. What prevented the formation of such a viewpoint

for a very long time was the fact that independent of their differences, QM and QFT shared the Lagrangian and the functional integral setting which led to overlooking the significant conceptual differences. Up to the present it is not uncommon to find the misleading terminology "relativistic quantum mechanics" instead of QFT; As shown in section 2, interacting relativistic QM is something else than QFT. It is precisely the result which causal localization has on the mutual coupling of all states with the same superselected quantum numbers which led us to refere to this characteristic property as "Murphy's law" and the related property "nuclear democracy". In QM on the other hand, one can freely couple or decouple channels as one wishes.

This extraordinary holistic nature of QFT is precisely what renders it more fundamental than QM. But this comes at a prize, since it also makes QFT less susceptible to quantum mechanical computational techniques. Single operator methods of QM as, the construction of a Hamiltonian in terms of fields lead to well-defined renormalized expectation values only through ill-defined infinite intermediate steps (cutting off integrals, introducing ad hoc regulators in order to enforce finiteness of integrals) and there is no guaranty that these manipulation preserve the Hilbert space structure of the theory. The intrinsic method which is solely based on localization[43] and a certain minimality principle has no infinity but in certain cases requires to introduce additional couplings which was not explicitly there at the beginning (or to nonrenormalizable theories with infinitely many parameters). The modern era of QFT as a setting in QT which comes with its own set of concepts different from those of QM started at the end of the 50s with a programmatic talk by Rudolf Haag [59].

As explained in a previous subsection the foundational distinction of the two quantum theories is localization. Jordan, who after Einstein and Heisenberg was one of the most conceptual-philosophic motivated among 20th century physicists[44], left QFT shortly after the phenomenon of vacuum polarization was noticed in the context of "partial charge" (charge localized in a sphere [16]) in free field theories by Heisenberg and in interacting theories by Furry and Oppenheimer who noted that an interacting field applied to the vacuum create a one particle state accompanied by an infinite vacuum polarization cloud consisting of particle-antiparticle pair states. These were the first indications that there was a dramatic difference to QM. It is precisely these interaction-caused polarization clouds which determine the ultraviolet behavior of QFT and which, if not properly treated lead to ultraviolet divergences dealt with in renormalization theory. We know nowadays that localization causes thermal behavior including "localization entropy" [16][20]. It also causes all operators to communicate with all states having the same superselected charges; the Murphy's law leading to "nuclear democracy" without which Jordan's idea of a d=1+3 photon theory of neutrinos cannot even be formulated.

We remind the reader that the Einstein-Jordan conundrum from section 1

[43] The most prominent renormalization theory of this kind is the Epstein-Glaser setting.

[44] Among the early books on QFT his "anschauliche Quantentheorie"[76] is certainly the only one which dedicates entire chapers to the problem of finding a new conceptual-philosophical setting for interpreting the new theory.

has reappeared here because it shares with the bosonization/fermioniszation issue the same QFT formalism of chiral currents (25). But whereas the fluctuation conundrum has (less simple) extensions to higher no-conformal spacetime models, there is, as mentioned before no conceptual support from QFT for Jordan's use of two-dimensional bosonization/fermionization model as a realistic neutrino theory of light. Whereas typical quantum mechanical objects as oscillators can be adjusted to arbitrary dimensions (which justifies a pedagogical presentation in one spatial dimension), this is not the case in QFT. A Wigner particle as the photon and its free field is an object in d=1+3; Wigner's representation theory of the Poincaré group and the free QFT theory following from it in a functorial way depends on d=1+3, only there the characterization in terms wave functions and covariant Maxwell fields describes photons. But if one wants to criticize Jordan on this point, one has to include all his contemporaries (including his deriders) since all of them believed that a check of a structural property in the context of lowest possible dimension is enough for the validity in higher dimensions. Indeed none of them criticized him on dimensional grounds, rather those who articulated themselves as Fock [69] thought that he made a computational mistake[45].

There is an interesting connection of the discrete "Paulion" formalism which appears in the famous Jordan-Wigner paper [75] and the fermionization (24) which for generic charge $\alpha$ is really an "anyonization". The Jordan-Wigner transformation formalism becomes more concrete in d=1+1 when the abstract ordering passes to a concrete linear ordering. In the continuous limit one obtains exponentials of line integrals which make the nonlocal character of the "anyonic" (braid group) commutation relation explicit (related to the "fermionization"). Jordan has various footnotes [65] to the Jordan-Wigner paper but he dids not elaborate the connection between the two formalisms.

Jordan's model (24) has an interesting relation with the later Schwinger model [77]. The latter is a 2-dimensional massless quantum electrodynamics which Schwinger proposed in order to illustrate that a gauge theory is not necessarily describing photons and free electric charges, rather its observable content under certain circumstances may consist of massive vectormesons and screened electric charges free of any charge selection rule. In fact the gauge invariant content of the Schwinger model is described by a field which is the exponential of a massive free field i.e. it is of the form (24) except the field $\Phi_{schw}$ is now a massive[46] free field in d=1+1 which depends on space and time and not just on the lightray combination u=t+x. The model looses its screening aspect for short distances when

$$e^{i\alpha\Phi_{schw}(x,t)} \overset{s.d.}{\to} e^{i\alpha(\Phi(u)+\Phi(v))},\ u=t+x,\ v=t-x \tag{28}$$

The short distance limit is carried out on the n-point correlation functions by scaling the fields with the right mass power in such a way that no correlation di-

[45] Not so uncommon, before I red Klaiber [68] I also sensed that there was something wrong with this two-dimensional formalism.

[46] The mass is actually proportional to the square of the coupling strength.

verges. Then there are many n-point function which go to zero namely all those which violate the charge superselection rule (26); this is how the charge superselection emerges. Without this prescription the correlation functions would not belong to a Hilbert space. It turns out that the short distance limit of the massive Schwinger model is precisely the massless Jordan model which with a lot of imagination one could see as charge liberation (the Jordan model) in the short distance limit of the charge screened $QED_2$ (the Schwinger model).

This kind of screening is the two dimensional analog of the Schwinger-Higgs screening. The situation changes in case of n-component *massive* fermions; it has been suggested that these models describe the two-dimensional analog [71] of the still unsolved problem of quark confinement in 4-dimensional quantum chromodynamics (QCD). The liberation of charges in the asymptotic short distance region of the Schwinger model corresponds to the perturbative established asymptotic freedom in the nonabelian gauge description of QCD. Such a simple illustration of screening/confinement versus short distance charge liberation is only possible in d=1+1; free massless fields in higher dimensions do not permit such constructions. Hence although the Jordan model, different from the intentions of its protagonist, has no bearing on a neutrino theory of light (for whose validity there is not the slightest hint within the Standard Model, which is our presently best particle theory), it is believed to serve as a useful analogy for important unsolved problem of quantum chromodynamics which is the conundrum of quark confinement. A recent description of the Jordan model and its appearance in the massless limit of the Schwinger model can be found in [78].

The fermionization formula (24) of the Jordan model describes a chiral fermion only for one value of $\alpha$ whose square $\alpha^2$ corresponds (in a suitable normalization of the current) to the operator dimension $\dim\Psi = 1/2$[47]. For this value the $\Psi$ can be written as a Fourier transform in terms of Wigner particle creation and annihilation operators $a^*(p), a(p)$ and their charge-conjugate antiparticles. For other values of $\alpha$ one encounters a fields whose commutation relations are those of an anyon i.e. an object with commutation relations which go beyond the Boson/Fermion alternative and are related to a representation of the braid group. Such fields were proposed as a model of an "infraparticle" in the beginning 60s [79] which is a particle-like object which is behind the breakdown of standard scattering theory (the Bloch-Nordsiek infrared divergence problem). The reason is that for those charge values $\alpha \neq$ semiinteger the pointlike field description was lost in favor of an infinite stringlike behavior as in (24). The model on which this was first noticed, was a two-dimensional massive Dirac field coupled to the derivative of a massless scalar field. The long range interaction leading to a solution of the form

$$\psi(x) = \psi(x)_0 e^{ia\varphi(x)} = \psi(x)_0 e^{ia\int_{-\infty}^{x} \partial_\mu \varphi(x) dx^\mu} \tag{29}$$

Here $\psi_0$ is massive free Dirac field and since the infrared divergent zero mass scalar field is better interpreted as a string localized free field we prefer the

[47] The normalizations used in Jordan's work is different from that used in more recent times. We found it convenient to state the results without fixing normalizations.

second representation. The momentum space manifestation of the stringlike localization shows an interesting modification at the mass shell where in all theories describing particles there would be a $\delta(\kappa^2 - m^2)$ function. In terms of the Kallen-Lehmann spectral function the infraparticle spectral density for a massive d=1+1 infraparticle is instead

$$\left\langle \psi(x)\overline{\psi(y)} \right\rangle = \frac{1}{2\pi}\int d\kappa^2 \rho(\kappa^2)\int \frac{dp}{2\sqrt{p^2+\kappa^2}} p^\mu \gamma_\mu e^{ip(x-y)} \tag{30}$$
$$\rho(\kappa^2) = c(\alpha)\theta(\kappa^2 - m^2)(\kappa^2 - m^2)^{-d(a)}$$

The only important aspect of this formula is that a) for vanishing coupling strength $\alpha$ the infraparticle power law of the spectral density passes into the free field result $\rho(\kappa^2) = \delta(\kappa^2 - m^2)$ and b) it is not possible to write the spectral function as $\rho(\kappa^2) = \delta(\kappa^2 - m^2) + rest$ without violating the positivity of the rest i.e. In an infraparticle Hilbert space there is no possibility to sneak in a particle through the back door.

The infrared behavior of the perturbative treatment of this model are reminiscent of those encountered in QED which led to the strong suspicion that the minimal interaction of electron/positrons with photons is so strong in the infrared (for long distances) that the structure of the particle itself is affected i.e. the changes are more radical (see next section) than in the Coulomb interaction in QM where the modification of time dependent scattering theory leaves the structure of particle unaffected.

The study of chiral conformal QFT started in the beginning of the 70s with the Jordan model but without the knowledge about Jordan's work since the title under which he published it did not sound trustworthy. All the global conformal block decomposition results involving the universal conformal covering representation were checked for the Jordan model. Only with the appearance of the family of minimal models by Belavin, Polyakov and Zamolodchikov it became clear that the prior structural work was not in vain, the expected richness of chiral conformal QFT really materialized.

By now the wealth of very nontrivial results is staggering, the world of chiral models is meanwhile under impressive mathematical control. This and the infinite family of massive factorizing models are the only families where existence proofs of models have been achieved. It seems that Jordan had the right conceptual-mathematical instinct in emphasizing these models in many of his publication.

The multi-component extension of the Jordan model

$$j_k(u) =: \psi_k^*(u)\psi_k(u) := \partial\Phi_k(u),\ k = 1, ...n \tag{31}$$
$$\Psi(u, \vec{\alpha}) = e^{i\sum_{k=1}^n \alpha_k \Phi_k(u)}$$

is the starting point of a rather simple family of models whose maximal local extensions are characterized by even lattices. Their representations are classified by the finite number of dual lattices and there is a finite number of selfdual lattices which are connected with finite exceptional groups, the largest being the so-called *moonshine group*.

Another application in which one quantizes $\Phi_k$ with a n-dimensional zero mode attached to a n-dimensional quantum mechanics $p_k, q_k$ $k = 1, ..n$ was used for the operator formulation of the dual resonance model which led to string theory.

The history of the Jordan model in conjunction with the Schwinger model reveals that conceptually rich models develop a life of their own and are even able to survive flawed reasons which served their original introduction. One can be sure that at the time of the "neutrino theory of light" mock song neither Jordan nor his satyric colleagues had any firm idea about what message this 2-dim. model was supposed to reveal. In the 30's the extraordinary subtle conceptual and mathematical problems posed by interacting QFT was not yet appreciated and the idea of studying soluble models as a kind of theoretical laboratory in order to learn something about the classification and construction of interacting particles was still in the distant future. After Jordan's series of papers on the neutrino theory of light there were several other authors who published papers under the heading of neutrino theory of light without even mentioning how this can achieved in the realistic d=1+3 case of physical photons and neutrinos. There were several authors who continued to publish articles on this 2-dim. model under the title "neutrino theory of light" without bothering how to get to light and neutrinos in d=1+3 QFT world. It remains somewhat incomprehensible why in none of these papers commented on the discrepancy between title and content.

# 6 Nonlocal gauge invariants and an algebraic monopole quantization

Jordan was the first who realized that the passing from the classical electrodynamic to its quantum counterpart brought about a loss of locality. More specifically besides the local observables which in the gauge theoretical setting are by definition the quantum counterparts of the classical (second kind) gauge invariants any physical charge-carrying object cannot be better localized than a spacelike semiinfinite string. That the algebra of all physical fields is larger than that generated by local observables is not exceptional, but that electric charge-carrying operators cannot be compactly localized is unusual in a theory which has the name "local" is unusual. It raises the question which structure is responsible that beyond charge neutral local observables the charge sectors of a theory are nonlocal in that strong sense. It can be shown that any renormalizable theory which a charged current is related to zero mass s=1 field strength through a Maxwell equation as in QED falls into this class.

The best localization for a charged generating field is that of a semiinfinite Dirac-Jordan-Mandelstam string (DJM) characterized *formally* by the well-

known expression

$$\Psi(x;e) = \text{"}\psi(x)e^{\int_0^\infty ie_{el}A^\mu(x+\lambda e)e_\mu d\lambda}\text{"} \tag{32}$$

$$\Phi(x,y;e) = \text{"}\psi(x)e^{\int_0^1 ie_{el}A^\mu(x+\lambda(x-y))(x-y)_\mu d\lambda}\bar{\psi}(y)\text{"} \tag{33}$$

Such objects involving line integrals over vectorpotentials or their bilocal counterparts with a connecting "gauge bridge" (33) appear already in Jordan's work on attempts to quantize in a gauge invariant manner [80]. As an ardent positivist and a fierce defender of Heisenberg's maxim to use observables throughout, he even tried to formulate the dynamics of quantum electrodynamics solely in terms of gauge invariant operators. As a similar attempt more than 20 years later by Stanley Mandelstam, the effort fell short of what the authors had expected.

There is however one very pretty fall-out of these attempts which is worth mentioning. Being impressed by Dirac's magnetic monopole quantization, but not by his too classically looking geometric method of presentation, Jordan published a very different algebraic operator derivation in the same year [57]. Three years later he returned to this topic, this time presenting his arguments with more details and some helpful drawings. The argument in both papers is based on the use of the above "bridged" bilocals $\Phi(x,x')$. Starting from the commutation relations

$$[\Phi(x,x'),\Phi(y,y')] = \delta(y-x')\Phi(x,y')e^{i\omega(x,y',x')} - \delta(x-y')\Phi(y,x')e^{i\omega(y,x',x)} \tag{34}$$

$$\omega(x,y,z) = magnetic\ flux\ through\ triangle$$

$$Jacobi\ identiy\ for\ \Phi's \curvearrowright e^{i\sum_{tetraheder}\omega} = 1$$

The last line denotes the application of the Jacobi identity to the bridged bilocals whose validity turns out to be equivalent to the magnetic flux through a tetrahedron being integer valued (in certain unities). A similar method for monopole quantization where the result also emerges from cohomological argument involving a tetrahedron has been proposed by Roman Jackiw [58].

The quotation marks in the above formulas (32) highlights their formal aspects. Since these objects are not belonging to the local gauge invariant operators which appear in the formalism of n$^{th}$ order renormalized perturbation theory, they have to be defined (and renormalized) by hand, a gruesome task which was carried out by Steinmann [81] who succeeded to attribute mathematical meaning to these expressions. This technical work is important because electrically charged fields cannot be better localized than along a semiinfinite spacelike string and this has radical implication for the associated charged particles.

It is fascinating and very informative to follow the idea of gauge invariant nonlocal semiinfinite string-localized fields and that of bridged bilocals a bit more through the history of particle physics. In an historically important paper, written about the same time as the appearance of the above string-localized fields in Jordan's work, Bloch and Nordsiek, using a simple model, argued that the scattering of photons off charged particles will lead to infrared divergencies

unless one treats the problem in the way they proposed in their model. After the renormalization theory of QED was understood, Yennie, Frautschi and Suura (YFS) showed that although the scattering amplitudes are infrared divergent the inclusive cross section for a specified photon resolution $\Delta$ not; the artificially introduced cutoff from the "virtual" photons in the Feynman amplitude compensates with that coming from the integration over the infrared tail of the real photons to be summed over up to $\Delta$.

From a pragmatic view the formalism of YFS (i.e. compensating two infrared divergencies against each other) would have been the end; a finite answer which agrees with the experimental data would signal for many physicists: mission accomplished. But Jordan and some of his contemporaries had a strong philosophical motivation and one can almost be sure that this kind of pragmatic reasoning would not have been the end of the infrared issue in QED. Rather it would have been very much in the spirit of Jordan to search for a deep connection between the formula for the string-localized generating fields describing charge-carrying fields (32) and charged "particles". The parenthesis is to indicate that electrically charged particles in QED are not particles in the usual (Wigner) sense of irreducible (m,s) representations of the Poincaré group[48]. In this way it becomes clear that the breakdown of the scattering theory is not just because the interaction is long ranged as the quantum mechanical Coulomb scattering where the breakdown of the standard scattering theory through the appaerance of a logarithmic phase factor does not have any consequences for the structure of single particle states. Rather in QED the very existence of particles is affected, electrically charged particles are "infraparticles" and even in case of a one particle state the best one can do is prepare such a state that no photon with energy larger than $\Delta$ can emerge from such a state where $\Delta$ can be arbitrarily small.

In the previous section the issue of infraparticles came up in connection with a hidden infinite stringlike localization aspect of Jordan's two-dimensional model. In that case the two-point spectral function $\rho(\kappa^2)$ remains covariant. However in 4 dimensions the spacelike string direction defined in terms of a spacelike unit vector $e$ certainly enters the covariance low for $\rho$. The characteristic dissolution of the mass shell delta function into a singular cut, which starts at $p^2 = m^2$ is a general feature of infraparticles. Unitarity puts a lid on the strength of the singularity, it must be milder than a delta function. This leads to a vanishing in/out LSZ scattering limit; the perturbative infrared divergence of the scattering amplitude is a perturbative phenomenon; by summing up the leading terms and letting $\Delta \to 0$, the infinity is converted into zero.

Hence the pointlike gauge dependent Dirac spinor $\psi(x)$ which enters the Lagrangian is an unphysical chimera which has to be tolerated as an intermediate computational tool as long as one does not know how to formulate the dynamics and the computations directly in terms of physical operators. There is simply no compact localized operator which applied to the vacuum gener-

[48]The relativistic particle concept was laid down in Wigner's famous representation theoretical classification of 1939 [12].

ates a state with an electron or positron charge, only zero charge operators are compactly localizable. This is an illustration of the fact that the main and only physical principle is causal localization and its realization in Hilbert space (unitarity). Even the Lagrangian formalism and perturbation theory has to cede if it produces unphysical operators. The only known way up to know is to repair the perturbative situation by hand i.e. to construct the DJM object from the unphysical $\psi$ and $A_\mu$ which is a gruesome enterprise. The fact that QFT is governed by one principle, namely causal localization, does not make life simpler.

There is no doubt that the knowledge of the local observables in principle fixes the remaining physical operators, including bridged bilocals. But the question which remained unanswered since the time of Jordan is how, by what formula? If the theory would not be a gauge theory but one which only involves s=0 and s=1/2 fields it would be less problematic to construct bilocals from locals. The bilocal $:A(x)A(y):$ can be obtained from the product of two locals $:A^2(x):$ by a lightlike limiting procedure [82] and recently the problem whether bridged bilocals can be obtained in this way came up [83]. Having such zero charge bilocals, the DJM would be expected to result from the "shifting the (unwanted) charge behind the moon" argument.

There is however another more radical (but also more promising) method. The origin of all the problems goes back to Wigner's 1939 classification of positive energy representations of the Poincaré group. The transition from Wigner's form to the covariant form of the representation is not unique. In the massive case the (m,s) s=halfinteger Wigner representation can be described by infinitely many dotted/undotted spinorial fields for given spin s

$$\begin{aligned} &\psi^{(A,\dot{B})}(x),\ \ \left|A-\dot{B}\right| \leq s \leq A+\dot{B} \\ &m=0,\ s=\left|A-\dot{B}\right| \end{aligned} \tag{35}$$

The second line contains the formula for the significantly reduced number of zero mass spinorial wave functions; the popular potentials $A_\mu$ for s=1 and $g_{\mu\nu}$ for s=2 are not backed up by representation theory which would allow field strength ($F_{\mu\nu}$ for s=1, $R_{\mu\nu\kappa\lambda}$for s=2). In classical physics there is no such requirement because unitarity and Hilbert space requirement is no issue. If one uses pointlike covariant potentials for the formulation of interactions (the standard method in QED) then one has to rely on a quantum gauge formalism (Gupta-Bleuler or BRST) which allows a cohomological return to a physical subspace which does not contain charged particles.

But there is another way which is more physical. Its starting point is the realization that the full spinorial possibilities (35) can be recovered with semiinfinite string-localized covariant potentials $A_\mu(x,e), g_{\mu\nu}(x,e)$ $e=$ spacelike string direction. In this way the origin of the string-localized charged fields and their infraparticle behavior near the old mass shell are not surprising since the semiinfinite localization aspect enters through the vectorpotentials from the beginning. Although all the fields are now living in a physical Hilbert space, the Epstein-

Glaser iteration step is more complicated since the causal position of semiinfinite strings, which should preserve the string localization for counterterms, is now more involved. These new ideas may lead to the long overdue reformulation of gauge theory.

Closely connected with the string localization of the potentials is the Aharonov-Bohm effect. In fact there is a theorem which shows that behind this effect is the consequence of some basic structural difference between operator algebras generated by field strength associated to massless spin $s \geq 1$ as compared to their massive counterpart. Whereas for any compact simply connected spacetime region $\mathcal{O}$ there holds Haag duality

$$\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \tag{36}$$

i.e. the commutant of the algebra of $\mathcal{O}$-localized operators equals the algebra of operators localized in the causal complement of $\mathcal{O}$, if it comes to non simply connected regions (example toroidal spacetime regions $\mathcal{T}$) the $m = 0, s \geq 0$, the operator algebras show a violation [84]

$$\mathcal{A}(\mathcal{T})' \supset \mathcal{A}(\mathcal{T}') \tag{37}$$

of Haag duality which has no counterpart in the massive case. The explanation is that operator algebra $\mathcal{A}(\mathcal{T})$ generated by the field strength does not contain all physical operators, there are Bohm-Aharonov like nonlocal operators in $\mathcal{T}$ , but thanks to the breakdown of Haag duality the algebra of the field strengths "knows about its own imperfection". For higher spin the Bohm-Aharonov phenomenon has a generalization affecting also algebras localized in multifold connected regions. The use of stringlocalized potentials $A_\mu(x,e),\ g_{\mu\nu}(x,e), ...$makes this nonlocal aspect manifest [35] which remains hidden in the standard formalism and only suddenly pops out e.g. when one tries to construct electrically charged operators assuming that by some extension of perturbation theory one can get to such objects.

It is quite surprising that a theory as QED, which already in the middle of the 30s showed a rich conceptual structure, has still not reached its conceptual closure. This is even more so for its nonabelian extension the Yang Mills theory and quantum chromodynamics (QCD). We have gotten accustomed to nice words as "gluon" and "quark" confinement about which we think we know their content, but our understanding goes hardly[49] beyond the small subset of gauge invariant local operators. When it comes to the description of physical charge-carrying operators our formalism forsakes us not to mention the string localized counterparts of the DJM operators and the question what happened to the gluon and quark degrees of freedom.

Whereas the long stagnation on questions like this and on the standard model could be shrugged off by pointing to the complexity of the task, it is somewhat

[49]Whereas the vacuum correlations of gauge-invariant local observables are finite in the abelian case (and only on-shell formfactors and scattering amplitudes are infrared divergent ) in the nonabelian case also the gauge invariant correlation functions are divergent.

saddening to notice the loss of hard gained conceptional understanding. A typical illustration for the conceptual impoverishment is the story of infraparticles which begun with the infrared divergent scattering amplitudes and the recipe for calculating inclusive cross sections with a given inclusive resolution $\Delta$. As mentioned, the conceptual conquest of this problem started with the realization that the root of the problem is not merely a long range modification of scattering as in the quantum mechanics of Coulomb scattering, but a radical change of the particle concept. From Jordan's semiinfinite stringlike charged fields (32) to infraparticles is a long way. The first observation of a deviation from the standard particle structure was observed in a two-dimensional model [79] similar to Jordan's in the previous section. It was observed that the expansion of the cut which starts at the would be particle mass with respect to the coupling strength leads to similar terms as in the YFS work. The infraparticle aspect of electrically charged particles in QED was proven in the 80s, the most conceptual line of arguments in [85] established the infinite extension of electrically charged infraparticles as a consequence of the quantum adaptation of Gauss's law. At that point it became clear that the string-like extension is inexorably linked to the "dissolution" of the mass shell.

There are some examples in the history of particle physics where, as if led by an invisible hand, two separate discoveries which are different sides of the same coin are made at the same time but their internal connection is only seen many decades afterwards. Certainly the discovery of the infrared phenomenon in QED by Bloch and Nordsiek [86] and Jordan's semiinfinite spacelike localization of the physical charge generating operator in QED belongs to these cases. Being aware of this connection, it is not so surprising that the oldest mathematically controlled 2-dimensional models of infraparticles where constructed with the exponentials of zero mass field of the previous section, since the zero mass field in d=1+1 is really semiinfinite string (in this case halfline)-localized.

The loss of conceptional understanding in contemporary attempts to go beyond the standard (Wigner) particle concept becomes obvious in the present flood of papers on "unparticles", which appears as the particle equivalent of the German word "Unsinn". The authors owe an answer how their ill-defined objects can be placed into the conceptual quite dense meshwork of string-localized fields and its momentum space properties in terms of dissolved mass shells[50] Even in those few cases where they cite the infraparticle work, it is clear that they do not understand its conceptual basis. They seem to think that the YFS kind of infrared problem is similar to the infrared aspects of Coulomb scattering where the one-particle remain those of standard particles. There seems to be nobody of sufficient knowledge of QFT whom they would listen to. The new generation of referees have the same background and are unable to lift the state of arts to where it has been in the past. This makes the question of the causes behind this derailment relevant, but this article about backtracks to Pascual

[50] Apparently the unparticle followers believe that the infrared divergencies in the scattering theory of electrically charged particles represent (like the quantum mechanical Coulomb scattering) just a small modification of scattering theory and not a radical change of the particle concept.

Jordan is not the right setting to enter a critical analysis of the Zeitgeist.

# 7 An end of foundational QFT before its scientific closure?

The present work described Jordan's discovery of QFT in connection with a problem posed by Einstein's pro photon argument based on his fluctuation formula and his subsequent fruitful controversy with Dirac's particle-based viewpoint. It also tries to shed some light on Jordan's somewhat "futuristic" idea, formulated at the 1929 Kharkov conference with the hindsight of later developments in QFT which vindicated his rejection of a classical parallelism, or in his own words his search for an understanding "independent of classical crutches".

Such a conceptual setting was discovered 30 years later by Rudolf Haag at a time when, as the result of world war II and for other sociological reasons, the continuity of particle physics research was interrupted and a good part of the early history of QFT was forgotten. Haag sketched his project in which causal locality took the center stage for the first time at a conference in Lille 1958 [59], the first more detailed account [88] in which the causality principle consisted of the spacelike Einstein causality and the timelike causal shadow property (time slice property) extended by the closely related positive spectrum requirement. This formulation keeps all the important physical properties without using a quantization parallelism and is even independent of the chosen field coordinatization of local observables.

By removing Jordan's "classical crutches", LQP moves farther away from quasiclassical and perturbative approximations and turns more to toward issues which are structural in nature, owing to the fact that for a theory based on principles it is easier to draw structural conclusions than to classify and construct individual models within these defining principles. Thanks to the mathematically powerful formulation of causal locality in the form of modular localization during the last two decades, the first constructive results about the existence of nontrivial chiral models as well as existence proofs for some factorizing models mentioned in section 3.2 have emerged. With this modest success, LQP has entered a terrain which has been totally out of reach for standard QFT. Although not a bad start, such nontrivial models are test cases still belong to a "theoretical laboratory". Whereas two-dimensional models of QM capture the essential points, two dimensional models of QFT do not allow to make structural conclusions about realistic higher dimensional models.

As in real life, the frustration resulting from running against a wall in trying to solve a crucial problem leads to the invention of distractions in the form of palliatives, in the Lagrangian approach to QFT they took the form of: "effective QFT" and "living with infinities", to name a few. The new concept of modular localization can be viewed as retaking Jordan's attempt to get away from the parallelism to classical theories inherent in quantization and move towards an intrinsic description of QFT and approximations which preserve its holistic

localization structure.

This program could not have been pursued at the time of Jordan since the mathematical-conceptual foundation in this first phase after discovery was too weak. But the motivation to place QFT on a foundational course reappeared independently three decades later in the Haag and LSZ setting, where the subtle connection between fields and particles was studied and its origin in causal localization was identified. The profound understanding of these issues could however not prevent the occasional appearance of misunderstandings. An example without lasting consequences was the publication of an article in the prestigious Phys. Rev. Lett. suggesting that Fermi's arguments by which he wanted to show that the local velocity[51] in QED is c (as in its classical Maxwell counterpart) is incorrect [60]. This claim was echoed in an article by Maddox in Nature and picked up by the world press. Besides bringing short-lived fame for having removed theoretical obstacles against the existence of time machines, the net result of this affair was a strengthening of the modular localization of QFT [61]. This illustrates the positive side of committing a conceptually interesting error and afterwards resolving it with even more interesting subtle arguments.

In contrast the misunderstandings underlying the interpretation of the dual model and string theory were of a different caliber. One reason why they has been immune against critique is that it is not the proposal of an individual but of physicists embedded in a rather uncritical community, the precursor of the later globalized community. This is perhaps the most inappropriate social form for doing foundational research since globalized communities cultivate physical monocultures and have no critical breaks. Up to the beginning of the 80s most speculative ideas where put to critical tests usually by more established and prominent researchers. In a highly speculative area as particle theory critique is essential for keeping the balance. The golden years of particle theory and QFT in the years 1950-1975 were also the years of "Streitkultur" going with the names as Pauli, Lehmann, Jost, Källén, Landau and although in the new world the tradition is somewhat different, one may add names as Oppenheimer, Feynman, Schwinger etc.

The string culture did away with this, instead of individuals or small groups localized at one physics department there is now a large globalized community of people who have a similar scientific background and dedicate their knowledge to the promotion of the theory around which the community was formed. Those members who by there seniority would have played a critical role in earlier times, act now more like gurus i.e. if criticism (as that about the localization in string theory) comes up they will refudiate it and make sure that no doubts linger on. To keep morals high, they praise the string theory as the "gift of the 21st century to the 20th", "the only game in town" and similar pronouncements (whereas critical individuals outside the community think of it more as a undisposed relic which the 20th left to the 21th century). Similar arguments which led to the claim that the quantization of the Nambu-Goto Lagrangian describes a

[51] In a relativistic QM only the effective velocity (analog of the acoustic velocity) whereas the local velocity is infinite.

spacetime string brought the community to accept the claim that the superstring contains quantum gravity and hence is the millennium theory of everything (TOE), thus insuring its hegemony over the future of particle physics.

Actually these tendencies are in fact attributes of the Zeitgeist and not indicators of weakening intellectual capabilities. Ignoring critique and warnings, in the belief to be in the possession of a "theory of everything" is not different from the human faults which led to the crisis of the finance capitalism. These developments do not accidentally occur at the same time, rather they are connected by a strong sociological undercurrent.

This changed social environment is particular strongly felt in Germany, the country where almost 90 years ago QM and QFT began and where a successful balance between speculative innovations and their critical review was maintained for a long time. This continuity was interrupted when more recent times the strange idea gained ground that foundational research can be bred in well-financed exellency centers by coupling the salary to the showroom value of a person. Such a coupling of the scientific value and career of an individual to the uncritical fashions cultivated in a globalized community is detrimental for the course of science. Even though string theory has no credible relation to the experimental reality of high energy physics laboratories, the presence of string theorists is deemed necessary for maintaining the laboratories international reputation. It is clear that according to such criteria string theory is in a superior position, especially since its erronous handling of localization in relativistic QT occured on such a subtle issue of particle theory which is far beyond textbook QFT and out of reach for most physicists. Conceptual errors in times of monocultures of globalized communities without critical breaks are not like those errors commited by individuals in the time of Jordan (of which some were mentioned in this article); they were usually rapidly understood and sometimes their correction led to a more profound understanding as compared to a direct progress without the erronous sidestep. Pauli's famous "not even wrong" has to be understood in this context.

It is therefore not surprising that after Berlin in the 90s, Munich (the MPI) at the beginning of this decade, now also Goettingen will not have QFT in its Theoretical Physics department. The expected loss of its academic basis at the university of Hamburg will set the seal on a more than 80 year tradition of innovative QFT in Germany. The University of Hamburg was for a long time the home of the renaissance of QFT after world war II (Lehmann, Haag); even though one chair went to string theory it still maintained its high level of quantum field theoretic research which it already had right after its foundations with Pauli's presence in the 20s[52]; in fact his famous epiphany which led to the exclusion principle, occurred in Hamburg, next to Göttingen one of the great centers of the historical quantum dialogue at the beginning of QT.

Many particle theorists hope that the results of the LHC experiments will help to get out of the present already more than 30 years lasting stagnation.

[52]Wolfgang Pauli as well as his senior Wilhelm Lenz, were products of Sommerfeld's creative hotbed of the new quantum physics in Munich.

It is difficult to imagine how this can happen in a situation where some of the most fashionable theoretical ideas (those leading to extra dimensions) originated from misunderstandings. Experiments cannot correct flawed theoretical ideas, and the experimental falsification of an incorrect theory is epistemologically void. the role of experiments is to exclude mathematially and conceptually correct models and to narrow the new search by continuing this process with respect to the reduced set of remaining models.

With the discontinuation of foundational QFT research groups around chairs at theoretical physics departments of German universities in the land in which Pascual Jordan discovered QFT in 1925 and which succeeded to recover from the intellectual losses of its Nazi past after world war II and again became a leading place for foundational innovations in QFT, this role now seems to come to an end. This occurs at a time when new ideas promise to revolutionize this theory a third time. Research on QFT requires foundational knowledge and a long breath and therefore depends on on continuity as no other area in physics. Once interrupted for one generation it probably cannot be re-created; as a result the foundational research in QFT may end (unlike that in QM which is a foundationally complete theory) before it reached its conceptual closure.

# References


[1] J. Ehlers, D. Hoffmann, J. Renn (es.) "Pascual Jordan (1902-1980), Mainzer Symposium zum 100. Geburtag", MPIWG preprint 329, (2007)

[2] B. Schroer, *Pascual Jordan, biographical notes, his contributions to quantum mechanics and his role as a protagonist of quantum field theory*. pp. 47-68. In J. Ehlers, D. Hoffmann, J. Renn (es.) "Pascual Jordan (1902-1980), Mainzer Symposium zum 100. Geburtag", MPIWG preprint 329, (2007)

[3] W. D. Beiglboeck, *Ernst Pascual Jordan als Author wissenschaftlicher und allgemeinbildender Schriften* pp-145-206. In J. Ehlers, D. Hoffmann, J. Renn (es.) "Pascual Jordan (1902-1980), Mainzer Symposium zum 100. Geburtag", MPIWG preprint 329, (2007)

[4] M. Born, W. Heisenberg and P. Jordan, *Zur Quantenmechanik II*, Zeitschr. für Physik **35**, (1926) 557

[5] A. Duncan and M. Janssen, *Pascual Jordan's resolution of the conundrum of the wave-particle duality of light*, arXiv:0709.3812

[6] B. Schoer, *The Einstein-Jordan conundrum and its relation to ongoing foundational research in local quantum physics*, arXiv:1101.0569

[7] A. Einstein, *Zur Quantentheorie der Strahlung*, Physikalische Zeitschrift **18**, (1917) 121–128. English translation in: Van der Waerden, 1968, 63–77

[8] P. Jordan, *Zur Theorie der Quantenstrahlung*, Zeitschrift für Physik 30 (1924), 297

[9] A. Einstein, *Bemerkungen zu P. Jordans: Zur Theorie der Quantenstrahlung*, Zeitschrift für Physik **30**, (1925) 784

[10] S. Hollands and R. E. Wald, *Quantum Field Theory Is Not Merely Quantum Mechanics Applied to Low Energy Effective Degrees of Freedom,* General Relativity and Gravitation **36**, (2004) 2595-2603

[11] W. Heitler, *The quantum theory of radiation,* Clarendon Press, Oxford 1936

[12] E. P. Wigner, Ann. Math. **40**, (1939) 149

[13] S. Weinberg, *What is Quantum Field Theory and what did we think it is* ?, hep-th/9702027

[14] P. Jordan, *Kausalitaet und Statistik in der Modernen Physik*, Die Naturwissenschaften **5**, (1927) 9

[15] H. Halvorson, *Reeh-Schlieder defeats Newton-Wigner: on alternative localization schemes in quantum field theory*, Philosophy of Science **68**, (2001) 111-133

[16] B. Schroer, *Studies in History and Philosophy of Modern Physics* **41** (2010) 104–127, arXiv:0912.2874

[17] F. Coester and W. N. Polyzou, Phys. Rev. D **26**, (1982) 1348-1379 and references therein

[18] W. Heisenberg, *Über die mit der Entstehung von Materie aus Strahlung verknüpften Ladungsschwankungen*, Verhandlungen der Sächsischen Akademie der Wissenschaften zu Leipzig, **86**, (1934) 317-322

[19] W. H. Furry and J. R. Oppenheimer, *On the theory of the electron and positive*, Physical Review **45**, (1934) 245-262

[20] B. Schroer, *A critical look at 50 years particle theory from the perspective of the crossing property*, Found. Phys. (2011) 41: 204–241, arXiv:0906.2874

[21] B. Bakamijan and L. H. Thomas, *Relativistic particle dynamics II*, Physical Review **92**, (1953) 1300-1310

[22] F. Coester, *Scattering theory for relativistic particles*, Helvetica Physica Acta **38**, (1965) 7-28

[23] N. S. Sokolov, *Interacting relativistic particles*, Doklady Akad. Nauk USSR 233, (1977) 575-592

[24] W. N. Polyzou, *Equivalent Hamiltonians*, Journal of Mathematical Physics **43**, (2002) 6024-6038, arXiv:nucl-th/0201013

[25] S. Weinberg, *The Quantum Theory of Fields I*, Cambridge University Press 1995

[26] S. S. Schweber, *QED and the men who made it; Dyson, Feynman, Schwinger and Tomonaga*, Princeton University Press 1994

[27] R. Haag, *Local Quantum Physics*, Springer 1996

[28] H.-J. Borchers, *On revolutionalzing quantum field theory with Tomita's modular theory*, Journal of Mathematical Physics **41**, (2000) 3604-3873

[29] W. G. Unruh, *Notes on black hole evaporation*, Phys. Rev. **D14**, (1976) 870-892

[30] J. J. Bisognano and E. H. Wichmann, *On the duality condition for quantum fields*, Journal of Mathematical Physics **17**, (1976) 303-321

[31] G. L. Sewell, *PCT and gravitationally induced* , Annals of Physics **141**, (1982) 201-224

[32] B. Schroer, Nucl. Phys. B **499**, (1997) 447

[33] R. Brunetti, D. Guido and R. Longo, *Modular localization and Wigner particles*, Rev. Math. Phys. **14**, (2002) 759-785

[34] L. Fassarella and B. Schroer, *Wigner particle theory and local quantum physics*, J. Phys. A **35**, (2002) 9123-9164

[35] J. Mund, B. Schroer and J. Yngvason, Commun. Math. Phys. **268**, (2006) 621

[36] J. Mund, *Modular localization of massive particles with "any" spin in d=2+1 dimensions*, Journal of Mathethematical Physics **44**, (2003) 2037-2057

[37] R. Longo, Pacific Journal of Mathematics **70**, (1978), 199

[38] R. F. Streater and A. S. Wightman, PCT, *Spin and Statistics and all that*, New York, Benjamin 1964

[39] J. Mund, Annales Henri Poincare 2 (2001) 907-926

[40] B. Schroer, *Unexplored regions in QFT and the conceptual foundations of the Standard Model*, arXiv:1006.3543

[41] A. Connes, *Caracterisation des espaces vectoriels ordonees sousjacents aux algebres de von Neumann*, Ann. Inst. Fourier **24, (**1974**)** 121

[42] O. Zapata Marin, *On Facts in Superstring Theory. A Case Study: The AdS/CFT Correspondence*, arXiv:0905.1439, see also: Spinning the superweb, essays on the history of string theory, http://www.spinningthesuperweb.blogspot.com/

[43] K. Pohlmeyer, Phys. Lett. **119 B**, (1982) 100

[44] Y. Nambu, Lectures at the Copenhagen Symposium, 1970, unpublished, T. Goto, Progr. Theor. Phys. **46** (1971) 1560

[45] E. Martinec, Class. Quant. Grav. **10**, (1993) 187

[46] D. A. Lowe, Phys. Lett. B 326, (1994) 223

[47] P. Di Vecchia, *The birth of string theory*, arXiv 0704.0101

[48] E. Witten, *Global aspects of current algebra*, Nuclear Physics B 223 (1983) pp. 422–432

[49] Y. Kawahigashi, R. Longo, U. Pennig and K.-H. Rehren, *The classification of non-local chiral CFT with c<1*, Commun. Math. Phys. **271**, (2007) 375

[50] G. Lechner, *On the Construction of Quantum Field Theories with Factorizing S-Matrices*, PhD thesis, arXiv:math-ph/0611050

[51] G. Mack, *D-dimensional Conformal Field Theories with anomalous dimensions as Dual Resonance Models*, arXiv:0909.1024, *D-independent representations of conformal field theories in D dimensions via transformations to auxiliary dual resonance models. The scalar case*, arXiv:0907:2407

[52] N. N. Bogoliubov, A. Logunov, A. I. Oksak and I. T. Todorov, General principles of quantum field theory, Dordrecht Kluwer

[53] D. Buchholz and K. Fredenhagen, *Locality and the structure of particle states*, Commun. Math. Phys. **84**, (1982) 1-54

[54] P. Jordan, *The Present State of Quantum Electrodynamics*, in *Talks and Discussions of the Theoretical-Physical Conference in Kharkov* (May 19.-25., 1929) Physik.Zeitschr.XXX, (1929) 700

[55] P.Jordan and O. Klein, *Zum Mehrkörperproblem in der Quantentheorie*, Zeitschr. für Physik **45**, (1927) 75

[56] P. Jordan and W. Pauli, *Zur Quantenelektrodynamik ladungsfreier Felder*, Zeitschr. für Physik **47**, (1928) 151

[57] P. Jordan, *Zur Quantenelektrodynamik. III. Eichinvariante Quantelung und Diracsche Magnetpole*, Zeitschr. für Physik **97**, (1935) 535

[58] R. Jackiw, Int. J. Mod. Phys. A **19S1** (2004) 137

[59] R. Haag, *Discussion of the 'axioms' and the asymptotic properties of a local field theory with composite particles* (historical document), Eur. Phys. J. H 35, 243–253 (2010)

[60] G. C. Hegerfeldt, Causality problems in Fermi's two atom system, Phys. Rev. Lett. **72**, (1994) 596-599

[61] D. Buchholz and J. Yngvason, *There are no causality problems for Fermi's two atom system*, Physical Review Letters **73**, (1994) 613-616

[62] H. Epstein and V. Glaser, The role of locality in perturbation theory. Ann. Inst. H. Poincaré A**19** (1973), 211

[63] S. Doplicher and J. Roberts, Why is there a field algebra with compact gauge group describing the superselection structure in particle physics, Comm. Math. Phys. **131** (1990) 51

[64] B. Schroer, Modular Wedge Localization and the d=1+1 Formfactor Program, Ann. Phys. 295 (1999), 190–223

[65] P. Jordan, *Beitraege zur Neutrinotheorie des Lichts*, Zeitschr. fuer Physik **114**, (1937) 229 and earlier papers quoted therein

[66] B. Schroer, *Bondi-Metzner-Sachs symmetry, holography on null-surfaces and area proportionality of "light-slice" entropy*, to appear in Foundations of Physics, arXiv:0905.4435

[67] P. Jordan, *Die Neutrinotheorie des Lichts*, Zeitschr. für Physik **93**, (1935) 464

[68] B. Klaiber, *The Thirring Model*, in O. A. Barut and W. E. Brittin (eds.) Lectures in Theoretical Physics, Gordon and Breach, New York, volume 10A p. 141- 176

[69] V. Fock, Nature **136**, N3502 (1936) 1011

[70] J. Schwinger, Phys. Rev. Letters **3**, 296 (1959) 296

[71] L.V. Belvedere, J.A. Swieca K. D. Rothe and B. Schroer, Generalized Two-dimensional Abelian Gauge Theories and Confinement, Nucl. Phys, **B** 153 (1979) 112-140

[72] D. Buchholz, G. Mack and I. Todorov, Nucl. Phys. B (Proc. Suppl.) **5B** (1988) 20

[73] S. Doplicher and R. Longo, *Standard and split inclusions of von Neumann algebras*, Invent. Mat. **75**, (1984) 493-536

[74] A. Pais, *Inward Bound*, Clarendon Press, Oxford University Press 1986

[75] P. Jordan and E. Wigner, *Über das Paulische Äquivalenzverbot*, Zeitschr. für Physik **47**, (1928) 631

[76] P. Jordan, *anschauliche Quantentheorie*, Springer, Berlin 1936

[77] J. Schwinger, Phys. Rev. **128**, (1962) 2425; Theoretical Physics Trieste Lectures, 1962, p- 89, I.A.E.A., Vienna 1963

[78] B. Schroer, Annals Phys. **321** (2006) 435-479

[79] B. Schroer, Fortschr. Phys. **11,** (1963) 1, B. Schroer, *A note on infraparticles and unparticles*, arXiv:0804.3563

[80] P. Jordan, Zeitschrift fuer Physik **95**, (1935) 202

[81] O. Steinmann, Commun. Math. Phys. **87**, (1982) 259

[82] J. Langerholc and B. Schroer, Commun. Math. Phys. **4**, (1967) 123

[83] S. Jacobs, *Gauge bridges in classical field theory*, DESY-THESIS-2009-009, http://unith.desy.de/research/aqft/diploma_theses/

[84] P. Leyland, J. Roberts and D. Testard, Duality for Quantum Free Fields, unpublished notes , CNRS Marseille, (1978)

[85] D. Buchholz, Phys. Lett. B 174 (1986) 331

[86] F. Bloch and A. Nordsiek, Phys. Rev. **52**, (1934) 54

[87] O. Darrigol, *The origin of quantized matter fields*, Hist. Stud. Phys. Sci.16/2, 198

[88] R. Haag and B. Schroer, J. Math. Phys. **3**, (1962) 248